Large-scale structure

Lectures 1-2

Introduction and context

Observables

Linear matter power spectrum

Linear perturbation theory

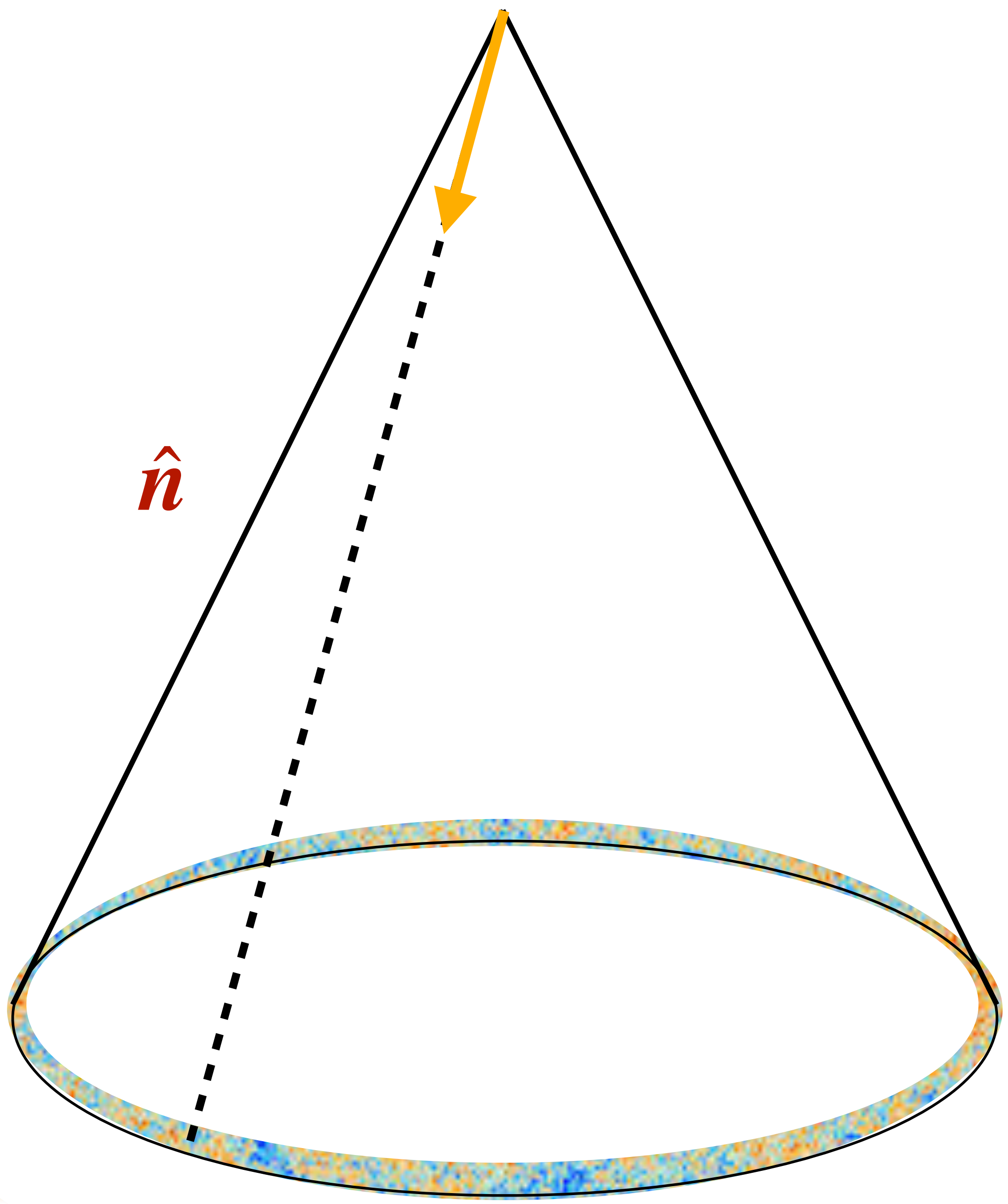
Redshift-space distortions

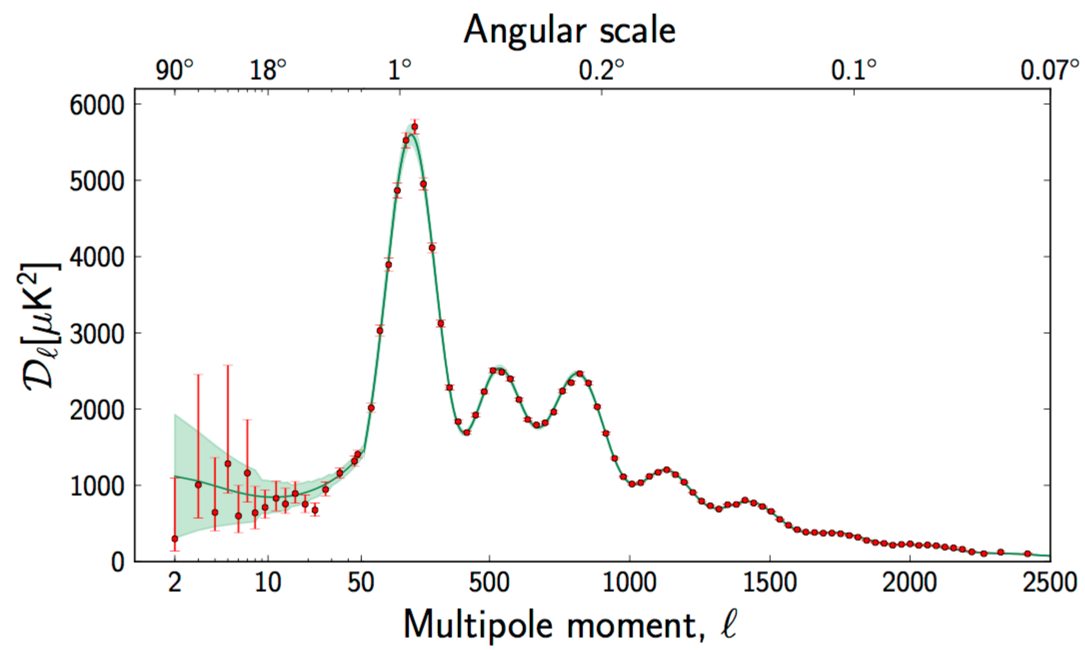
Linear galaxy bias

Linear galaxy power spectrum multipoles

Cosmic Microwave Background

CMB extremely successful. Better polarization in the next ~10 yrs



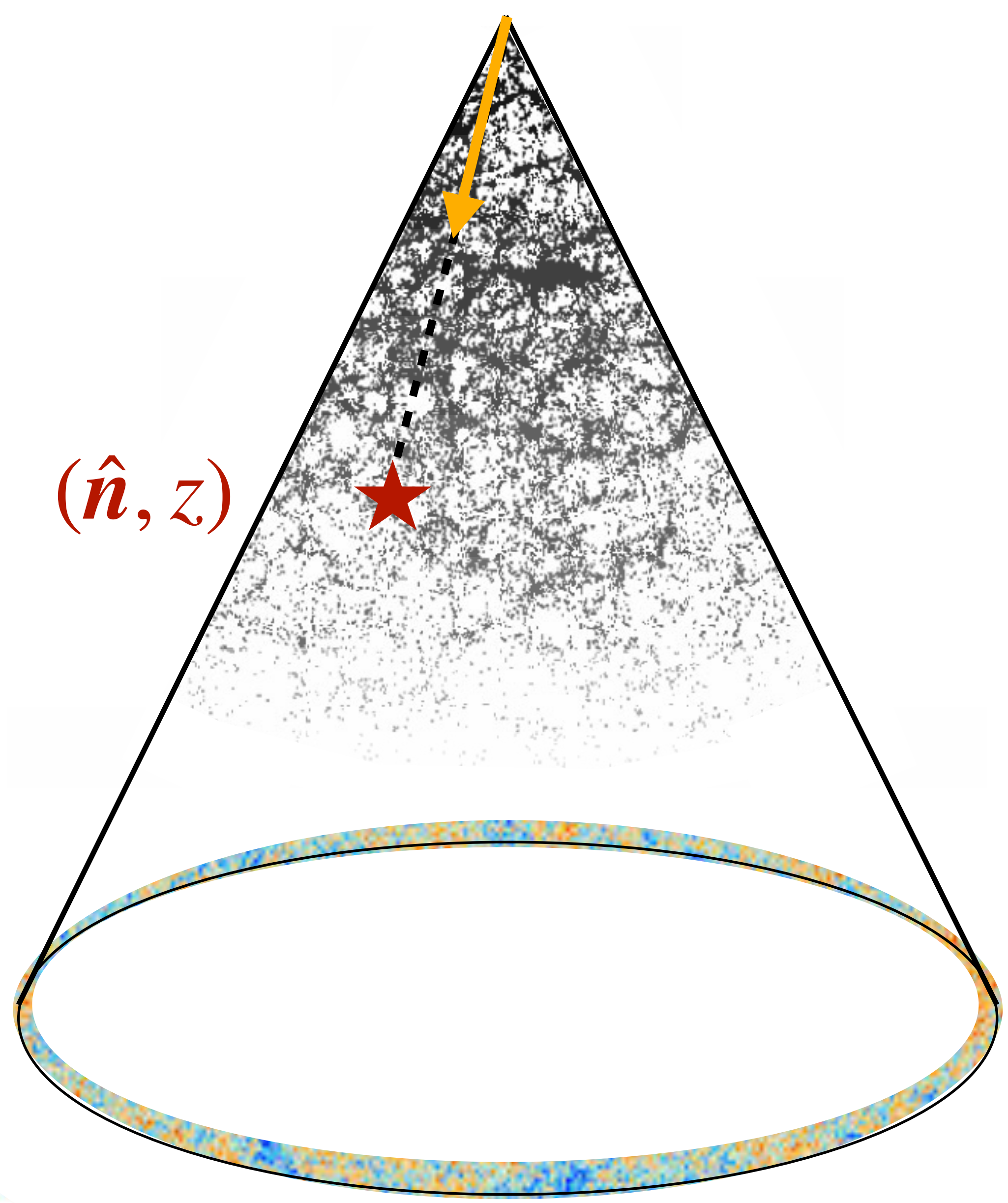


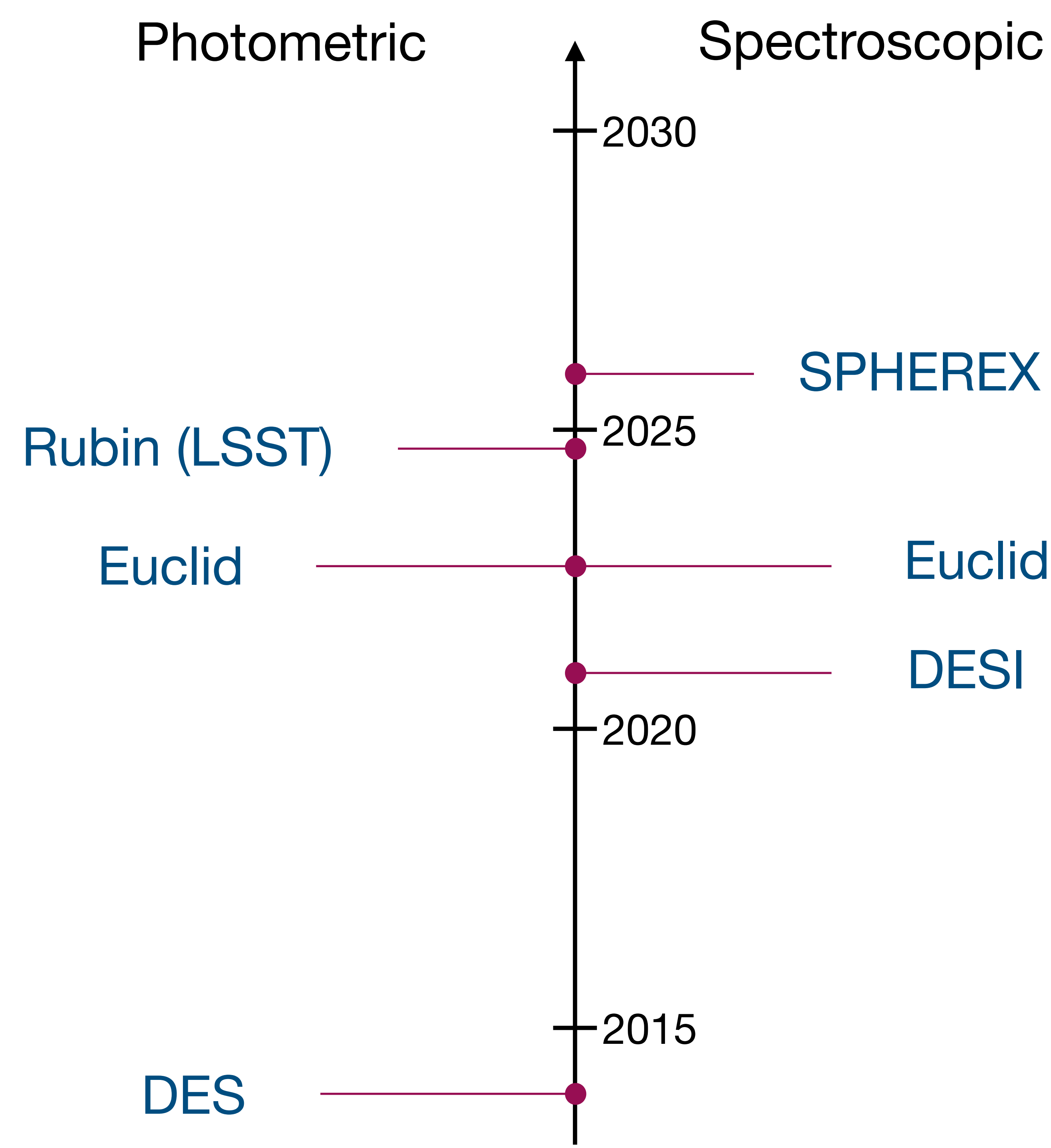
Parameter	*Planck* alone
$\Omega_b h^2$	0.02237 ± 0.00015
$\Omega_c h^2$	0.1200 ± 0.0012
$100\theta_{MC}$	1.04092 ± 0.00031
τ	0.0544 ± 0.0073
$\ln(10^{10} A_s)$	3.044 ± 0.014
n_s	0.9649 ± 0.0042
H_0	67.36 ± 0.54

Many open questions that CMB alone cannot answer!

Observing the entire light-cone

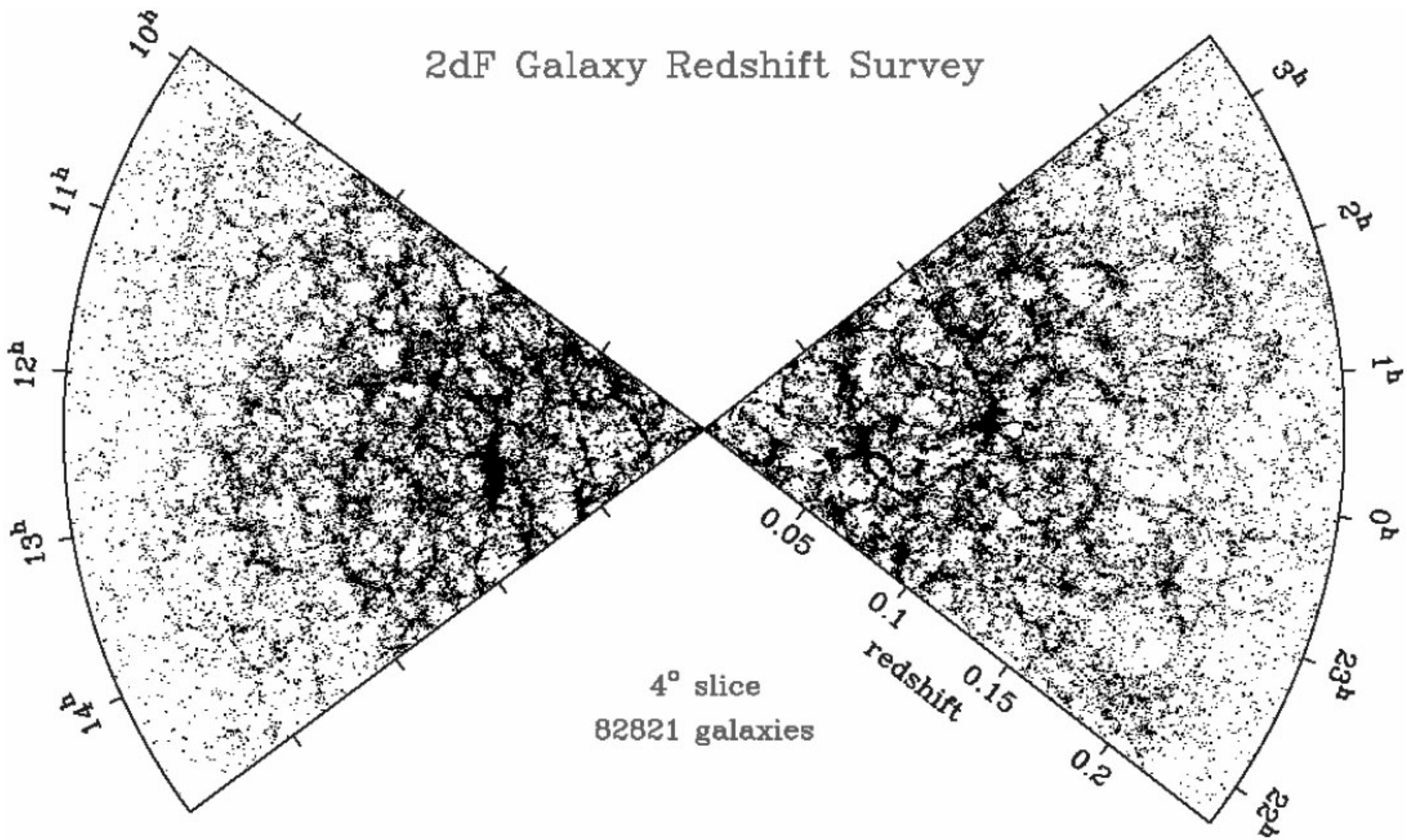
Image billions and take spectra of ~100 million of objects up to z<5



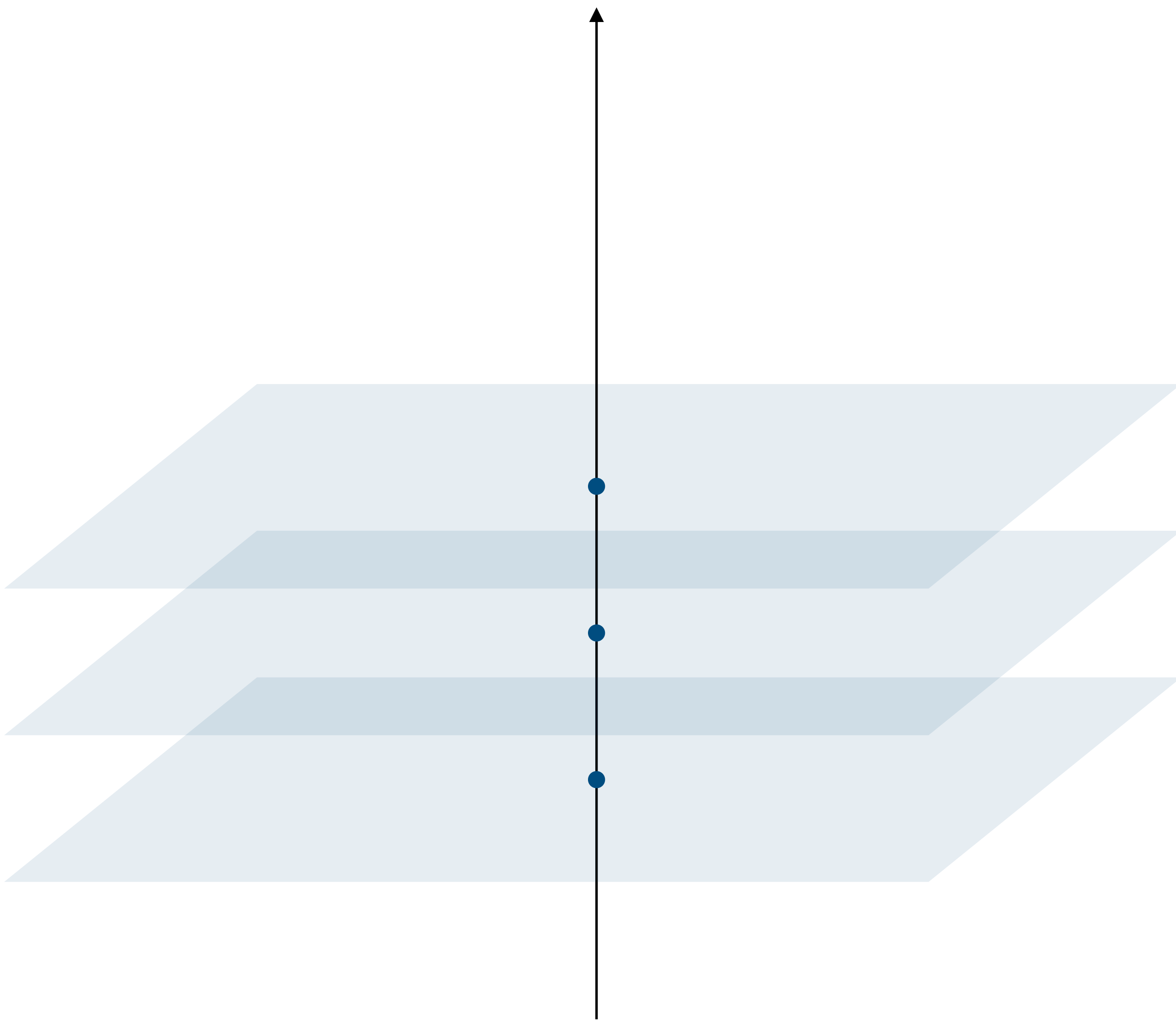


Large-scale structure

Structure in clustering of matter on large scales (larger than ~1Mpc)



Large-scale structure



$$\mathcal{O}(\boldsymbol{x},\tau) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\mathcal{O}(\boldsymbol{k},\tau)\, e^{i\boldsymbol{k}\boldsymbol{x}}$$

Observables are small fluctuations

In linear theory:

$$\mathcal{O}(\boldsymbol{k},\tau) = T_{\mathcal{O}}(k,\tau)\zeta(\boldsymbol{k})$$

primordial fluctuations
(initial conditions)

Large-scale structure

The correlation function and the power spectrum

$$\langle \mathcal{O}(\boldsymbol{x},\tau)\mathcal{O}(\boldsymbol{x}',\tau)\rangle = \xi(|\boldsymbol{x}-\boldsymbol{x}'|,\tau)$$

↓

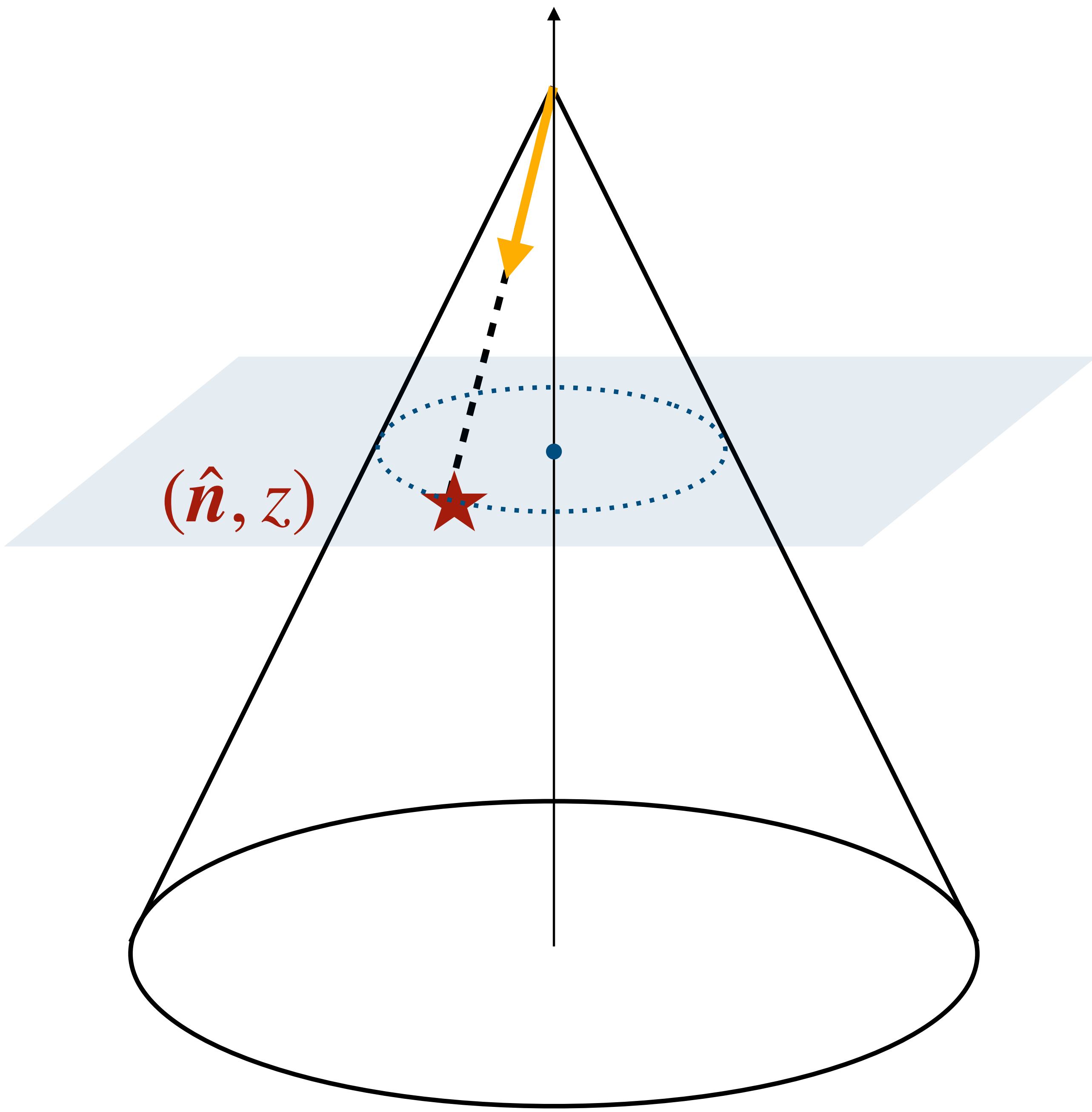
$$\langle \mathcal{O}(\boldsymbol{k},\tau)\mathcal{O}(\boldsymbol{k}',\tau)\rangle = (2\pi)^3\delta^D(\boldsymbol{k}+\boldsymbol{k}')P_{\mathcal{O}}(k,\tau)$$

$$P_{\mathcal{O}}(\boldsymbol{k},\tau) = T^2(k,\tau)P_\zeta(k)$$

$$P_\zeta(k) = \frac{A_s}{k^{3-(n_s-1)}}$$

nearly scale-invariant
nearly Gaussian
…

Observables in the late universe



The light-cone is 3D

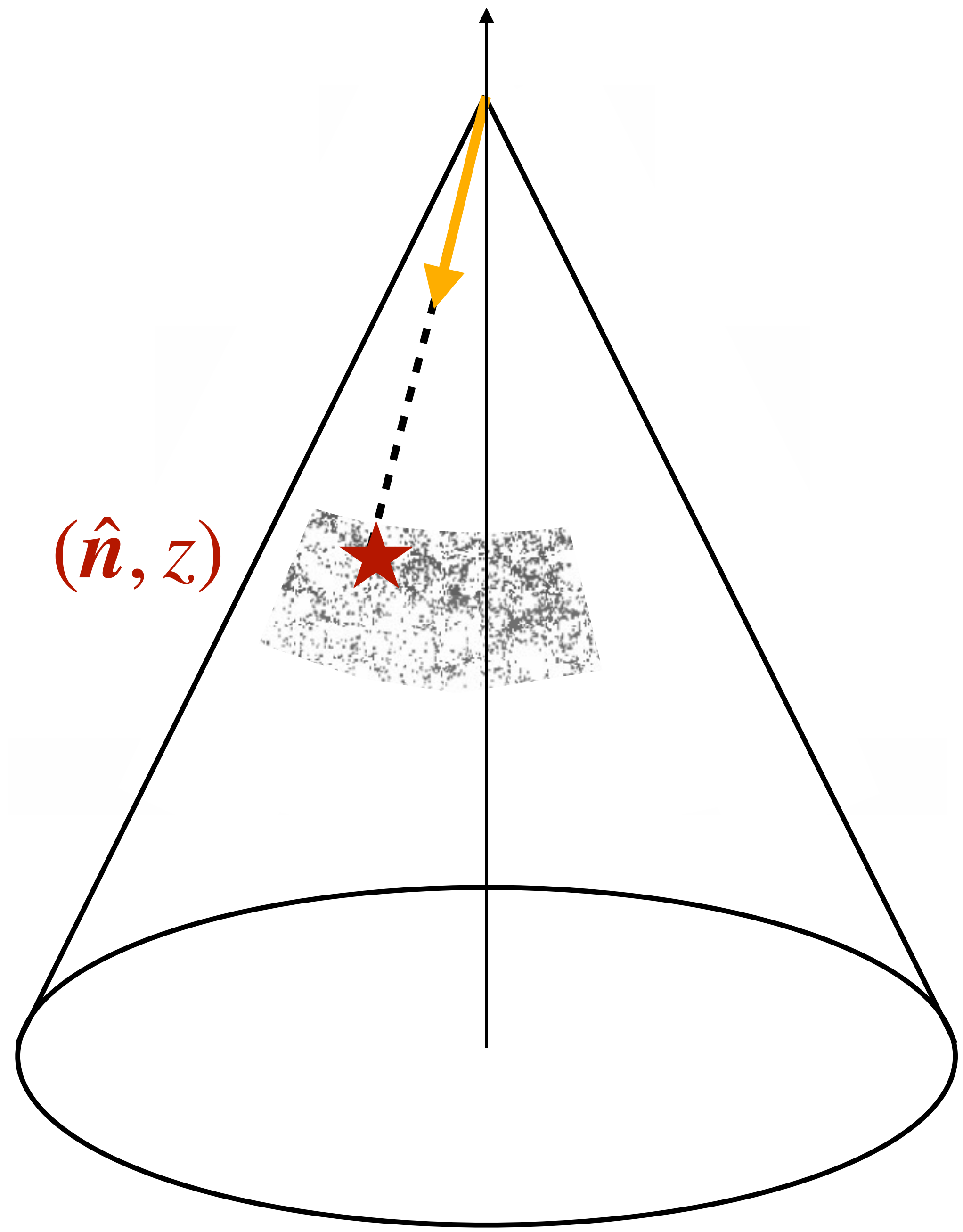
$$\mathcal{O}(\hat{\boldsymbol{n}}, z) = \sum_{\ell,m} \mathcal{O}_{\ell m}(z)\, Y_{\ell m}(\hat{\boldsymbol{n}})$$

$$\langle \mathcal{O}_{\ell,m}(z) \mathcal{O}_{\ell',m'}(z') \rangle = \delta^K_{\ell\ell'} \delta^K_{mm'} C_\ell(z, z')$$

One can project:

$$(\boldsymbol{x}, \tau) \rightarrow (\hat{\boldsymbol{n}}, z)$$

Observables in the late universe

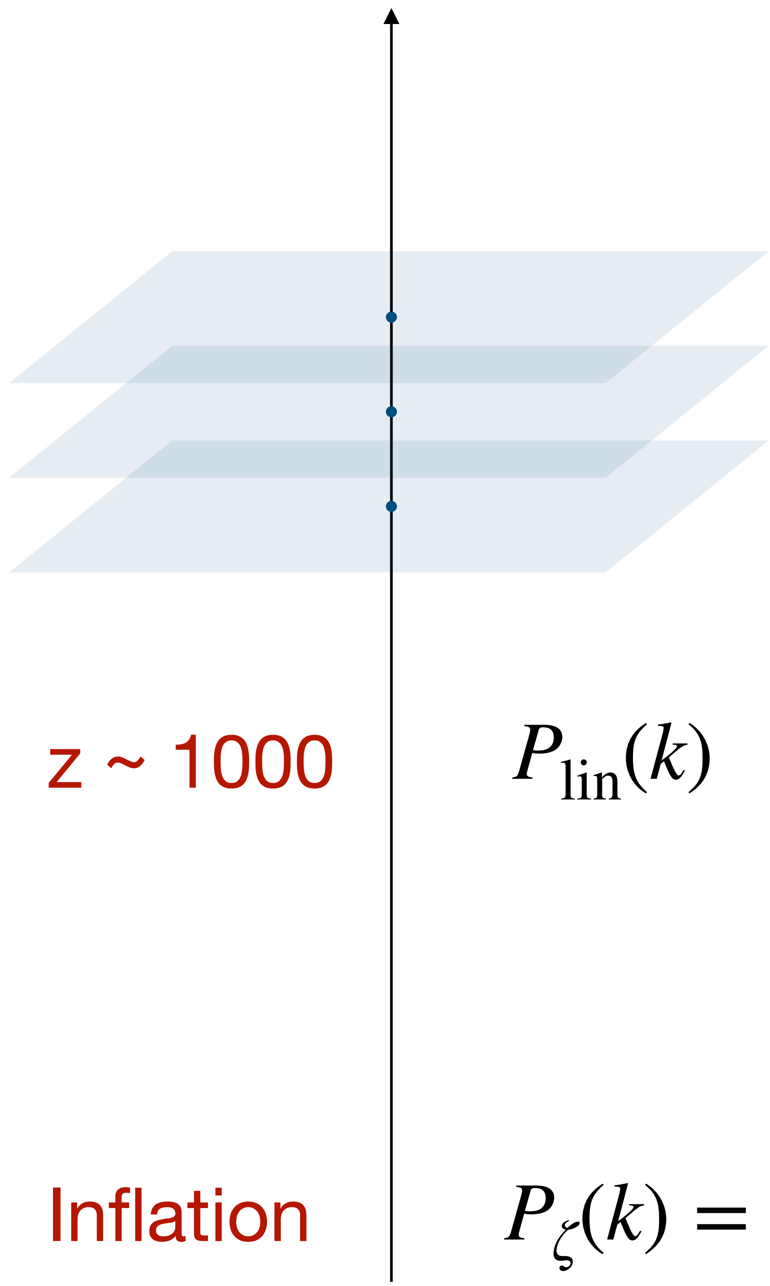


In a "small" patch $(\hat{\boldsymbol{n}}, z) \to \boldsymbol{x}$

In practice we mainly use $P_{\mathcal{O}}(k, \tau)$ for galaxies and C_ℓ for CMB

Full-sky, wide angle effects will be more important in the future

Linear matter power spectrum



$P_{\rm lin}(k)$

$$P_\zeta(k) = \frac{A_s}{k^{3-(n_s-1)}}$$

Matter fluctuations

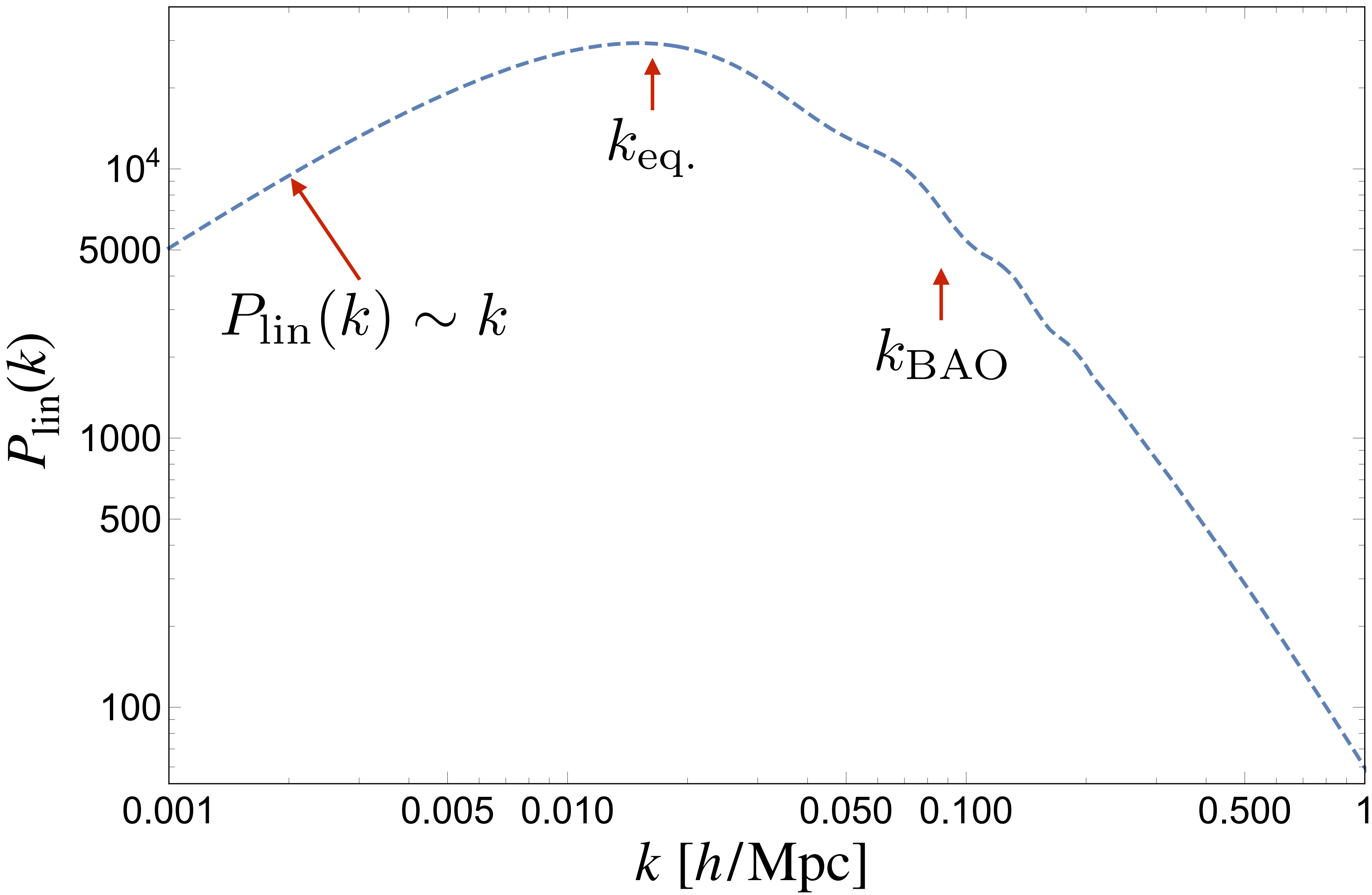
$$\delta(\boldsymbol{x}, \tau) = \frac{\rho(\boldsymbol{x}, \tau) - \bar{\rho}(\tau)}{\bar{\rho}(\tau)}$$

After recombination, neglecting GR

$$\langle \delta(\boldsymbol{k}) \delta(\boldsymbol{k}') \rangle = (2\pi)^3 \delta^D(\boldsymbol{k} + \boldsymbol{k}') P_{\rm lin}(k)$$

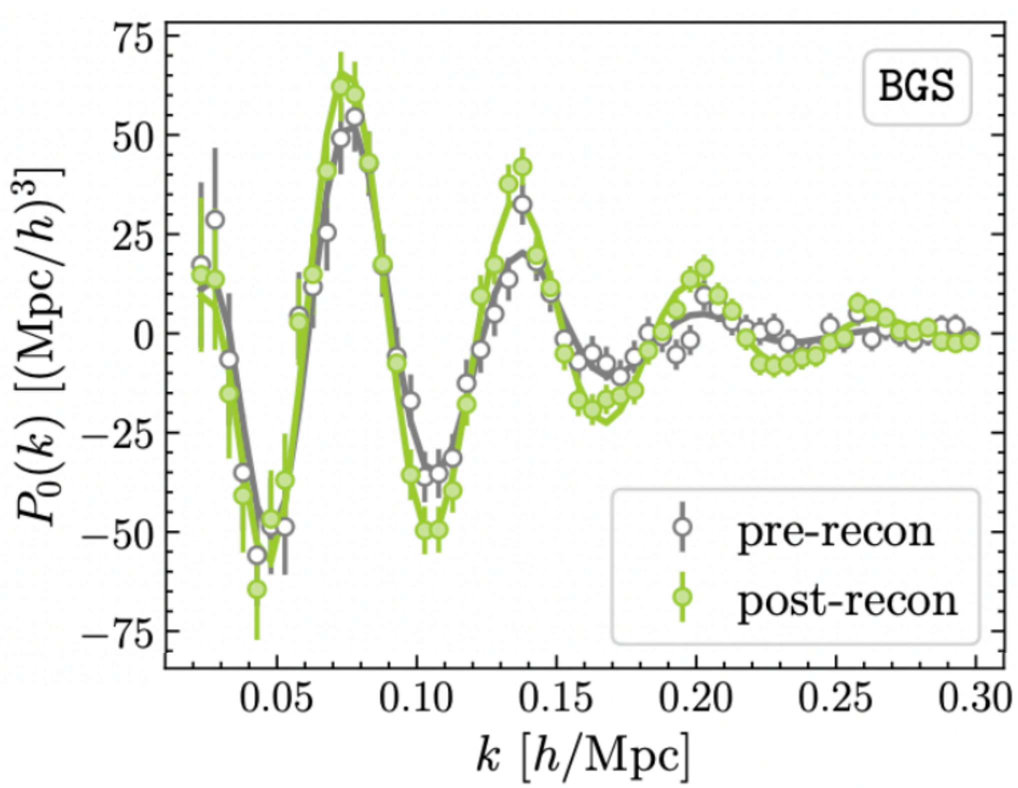
Linear matter power spectrum

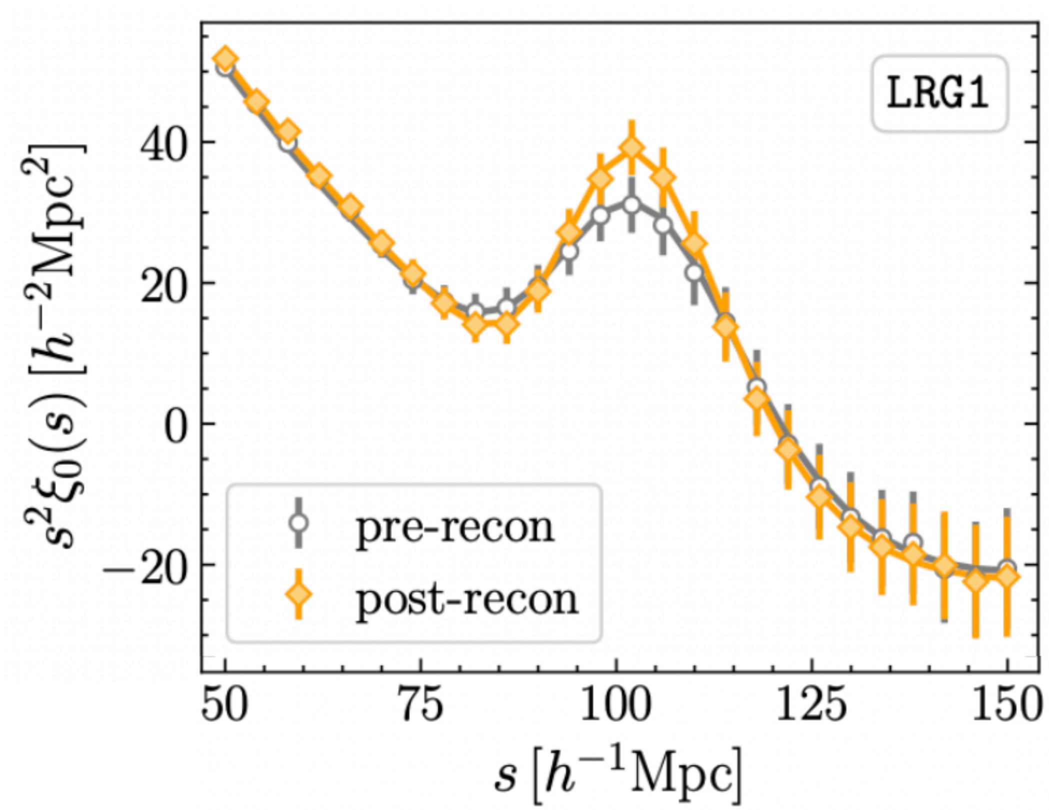
Main features of the linear power spectrum



Linear matter power spectrum

BAO in the correlation function looks like a single peak



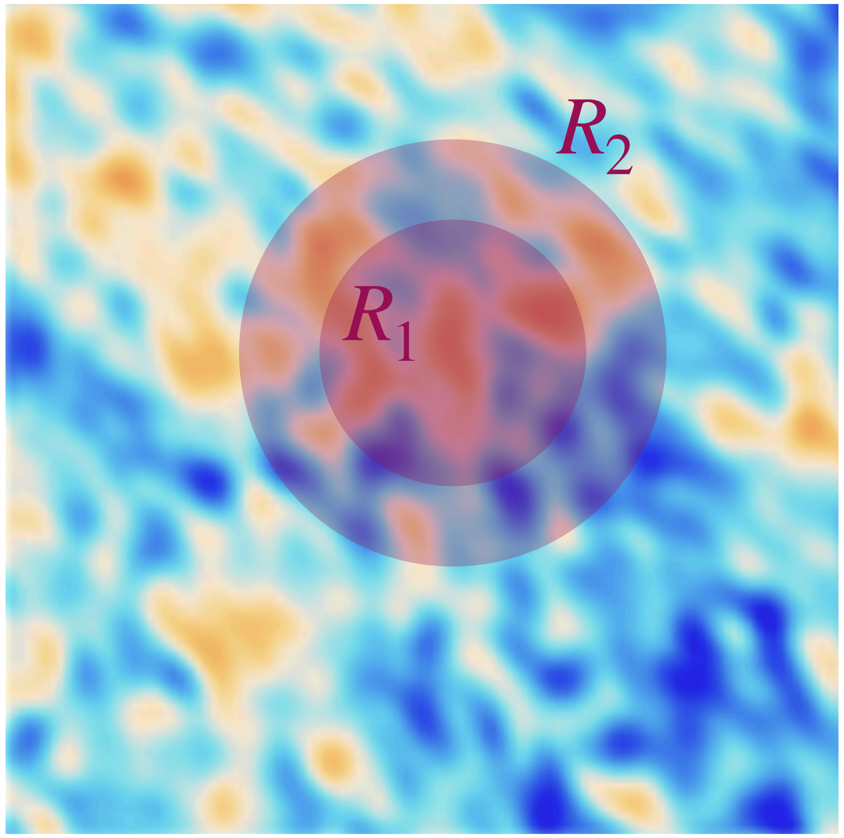


DESI 2024, adopted from Seshadri Nadathur

Linear matter power spectrum

Smooth the field on the scale R

$$\delta_R(\boldsymbol{x}) = \int d^3\boldsymbol{r} W_R(|\boldsymbol{x} - \boldsymbol{r}|)\delta(\boldsymbol{r})$$



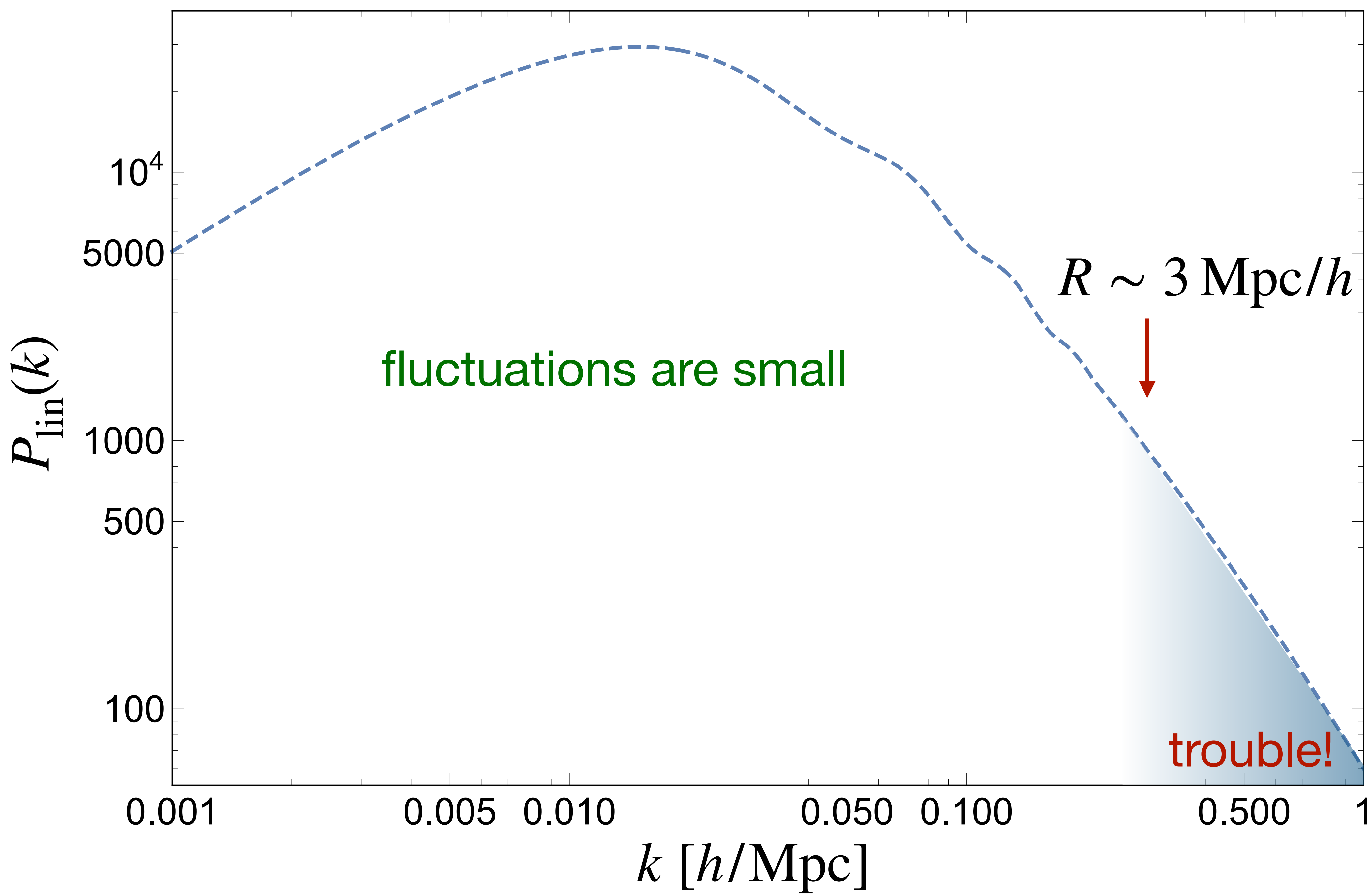
Variance of the smoothed density field

$$\langle \delta_R^2 \rangle \approx \int_{k<1/R} \frac{d^3\boldsymbol{k}}{(2\pi)^3} P_{\rm lin}(k)$$

$$\Delta^2(k) \equiv \frac{k^3}{2\pi^2} P_{\rm lin}(k)$$ → reduced (dimensionless) power spectrum

Linear matter power spectrum

At redshift zero, $\Delta^2(k) \approx 1$ for $k \approx 0.3\ h/\mathrm{Mpc}$



Linear perturbation theory

Linearized Einstein's equations in the non-relativistic limit

$$\delta' + \nabla_i v_i = 0$$

$$v_i' + \mathcal{H} v_i = -\nabla_i \Phi$$

$$\nabla^2 \Phi = 4\pi G \bar{\rho} a^2 \delta = \frac{3}{2}\Omega_m(\tau)\,\delta$$

These equations combine into

$$\delta'' + \mathcal{H}\delta' - \frac{3}{2}\Omega_m \delta = 0$$

$$\boxed{\delta(\boldsymbol{x},\tau) = D_+(\tau)\delta_0(\boldsymbol{x})} + D_-(\tau)\delta_0(\boldsymbol{x})$$

linear growth factor

for $\Omega_m = 1$, $D_+(\tau) = a$

Linear perturbation theory

One can also compute then the linear velocities

$$\delta' + \nabla_i v_i = 0$$

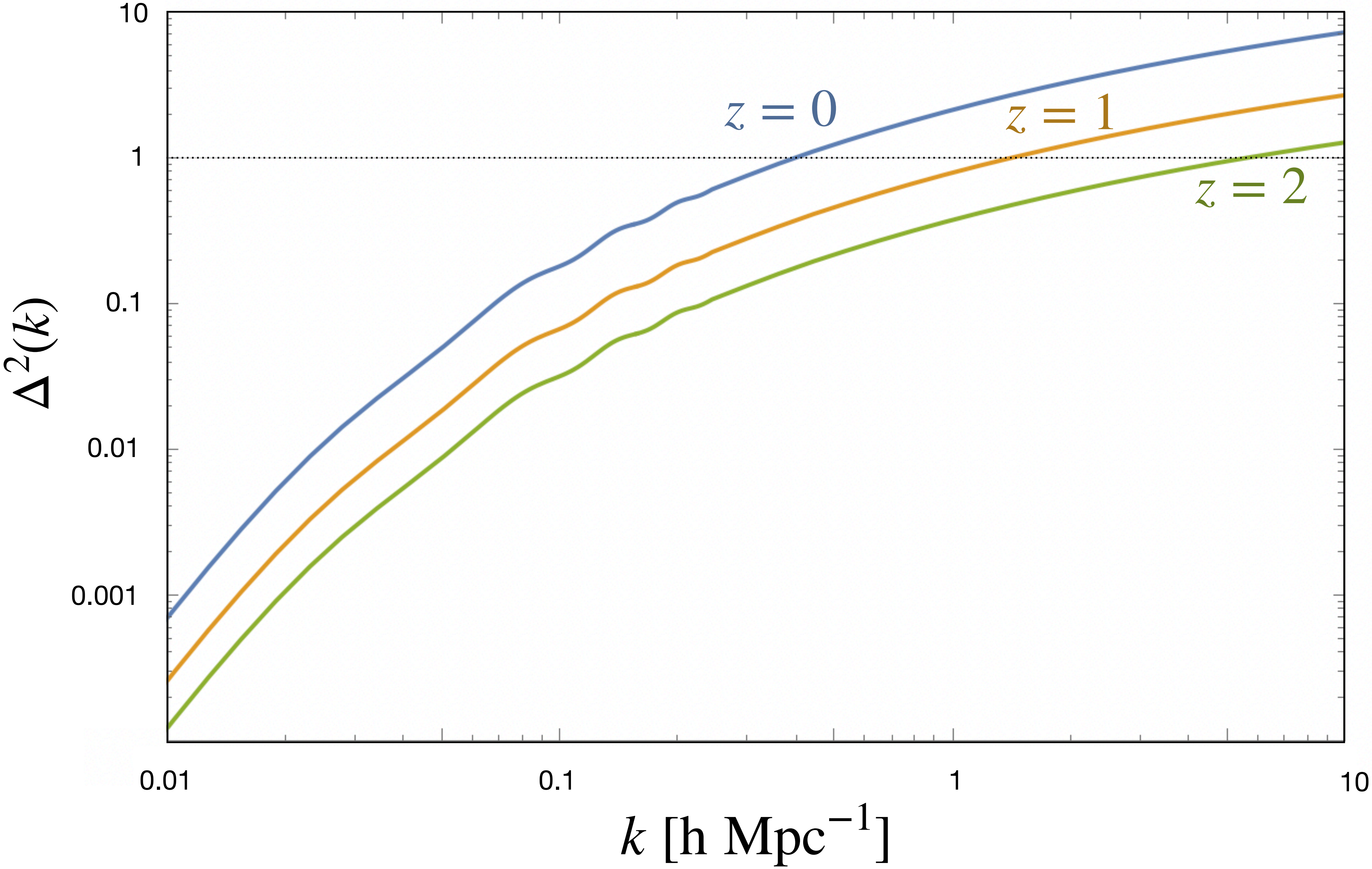
$$\delta' = \frac{da}{d\tau}\frac{d}{da}D_+(\tau)\delta_0 = \mathscr{H}\frac{d\log D_+}{d\log a}\delta$$

$$f(a) \equiv \frac{d\log D_+}{d\log a}$$

↑ growth function

$$v_i(\boldsymbol{k}) = if\mathscr{H}\frac{k_i}{k^2}\delta(\boldsymbol{k})$$

Linear perturbation theory



Linear perturbation theory

We do not observe the real positions, but redshifts

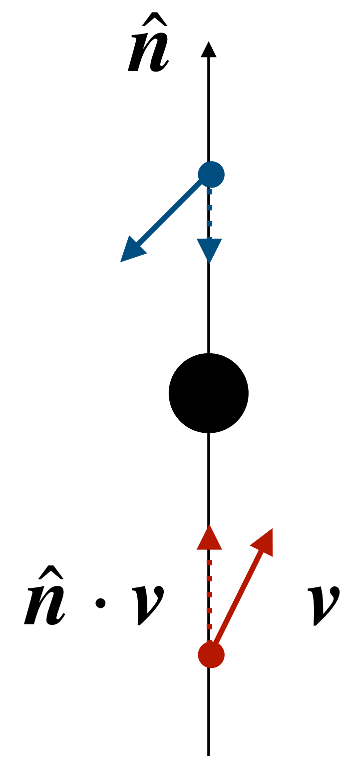
Mapping from redshift to real space introduces distortions

These are famous redshift space distortions

$$s_i = x_i + \frac{\hat{\boldsymbol{n}} \cdot \boldsymbol{v}}{\mathscr{H}} \hat{n}_i$$

$$v_i(\boldsymbol{k}) = i f \mathscr{H} \frac{k_i}{k^2} \delta(\boldsymbol{k})$$

$$\delta_s(\boldsymbol{k}) = (1 + f\mu^2)\delta(\boldsymbol{k}) \qquad \mu \equiv \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}}$$



Linear perturbation theory

The power spectrum is then

$$P_{\text{lin},s}(k,\mu) = (1 + f\mu^2)^2 P_{\text{lin}}(k)$$

It is convenient to expand this in multipoles

$$P_{\text{lin},s}(k,\mu) = \sum_{\ell} P_{\ell}(k)\mathscr{P}_{\ell}(\mu)$$

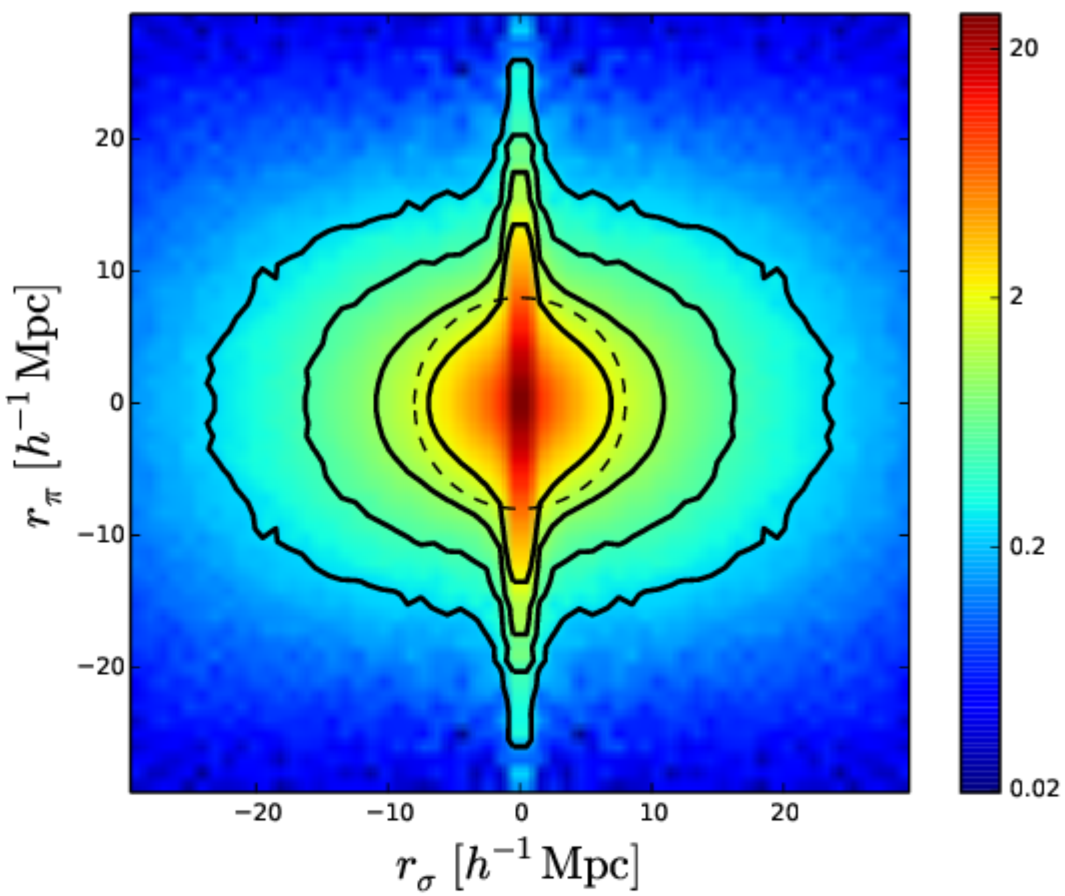
↑

power spectrum multipoles

$$P_0(k) = \left(1 + \frac{2}{3}f + \frac{1}{5}f^2\right) P_{\text{lin}}(k)$$

Linear perturbation theory

This is exactly what we measure from the data



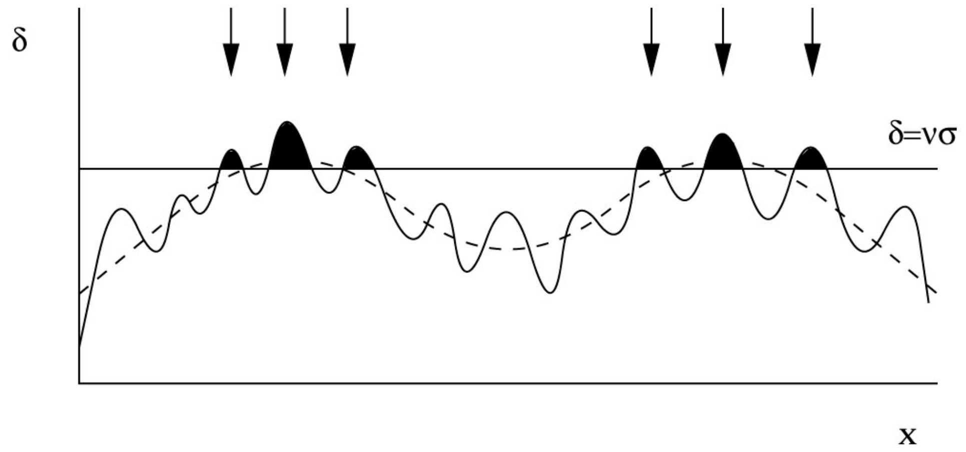
But to finally talk about galaxies we need one extra ingredient

Linear perturbation theory

Galaxies do not fairly represent dark matter

Naively, $n_g \sim n_{\mathrm{DM}}$ → $\delta_g = \delta$

But galaxies form only in sufficiently overdense regions!



On very large scales (in linear theory)

$$\delta_g = b_1 \delta + \cdots$$

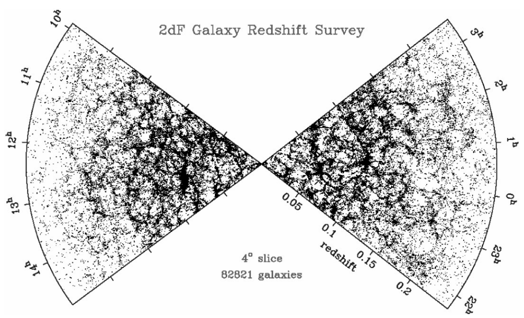
↑ galaxy bias

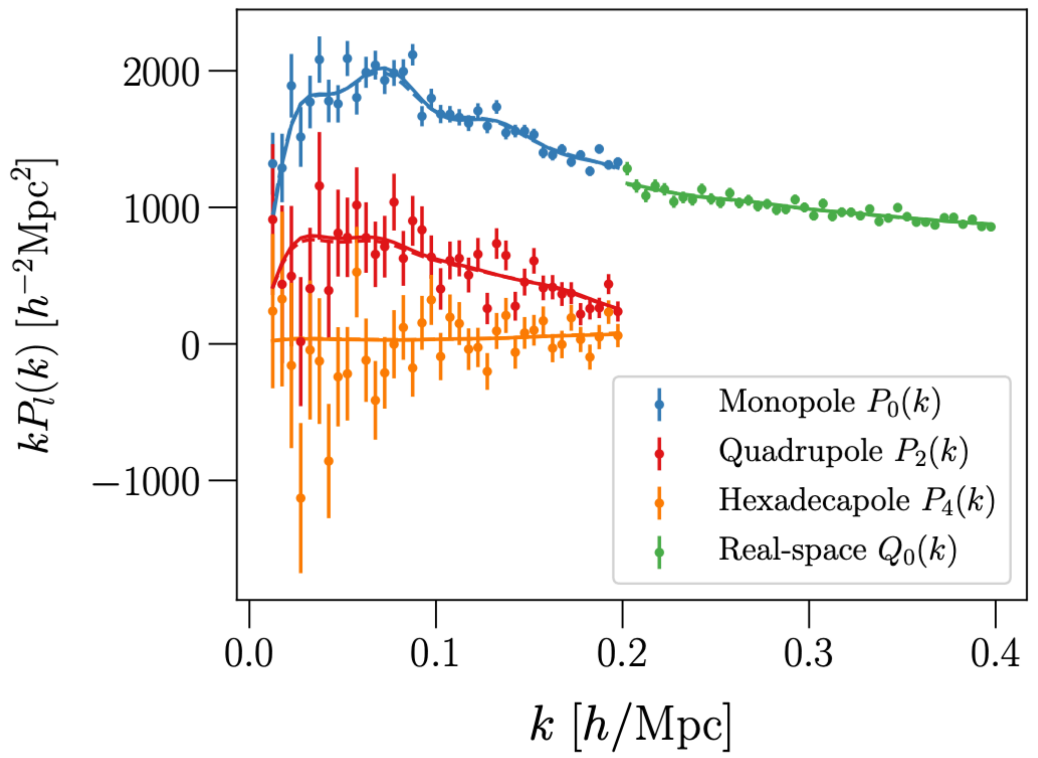
Linear perturbation theory

On large scales we get a famous Kaiser formula:

$$P_{g,\mathrm{lin},s}(k,\mu) = (b_1 + f\mu^2)^2 P_{\mathrm{lin}}(k)$$

galaxy map





Why is linear theory not enough? How to go beyond?