Problem set for the GW course, Petnica

Mehrdad Mirbabayi

1. In Einstein gravity, test particles moving in the field of a spherically symmetric star of mass M follow the geodisics of the Schwarzschild metric:

$$ds^2 = -(1 - \frac{r_g}{r})dt^2 + \frac{dr^2}{1 - \frac{r_g}{r}} + r^2 d\Omega^2 \tag{1}$$

   where $r_g = 2GM/c^2$.

   Calculate the small angle scattering (scattering angle χ in lowest order in $1/b$, b is the impact parameter) for an ultrarelativistic particle.

   In particular, calculate light deviation by the Sun, for a light ray just grazing the surface of the Sun (solar mass $M = 1.99 \times 10^{33}$ gr, radius $b = 700,000$ km, give your answer in arcminutes).

   What would be the bending angle if gravity was mediated by a scalar satisfying $\Box\varphi = 4\pi G T^\mu_\mu$?

2. Do you emit gravitational waves when you move your arm? (Hint: this problem does require a certain numerical estimate before you can tell.)

3. A body of mass $m \ll M$ passes by a body of mass M. In the small angle scattering approximation, estimate total radiated energy. Assume non-relativistic velocity.

4. Estimate how long it would take the Earth to fall down on the Sun due to emission of gravitational waves.

5. Estimate how far two black holes of mass $M = 30M_\odot$ rotating on a circular orbit can be, so that they merge within the life-time of the Universe (14 billion years) due to the emission of gravitational waves.

6. Consider a burst of GWs of the form $h_{yy} = -h_{zz} = A\cos(\omega(t - x))$ shining on LIGO mirrors, perfectly free masses in y direction, initially separated by $\Delta y = L \ll c/\omega$. Calculate the time it takes, measured by the clock attached to one of the mirrors, for the laser light to go to the other mirror and come back.