A gentle introduction to dark matter

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I. INTRODUCTION, EVIDENCE AND BASIC PROPERTIES OF DARK MATTER

First (and historical) notion of "dark matter" (DM) to get accustomed to:

> *In a number of astrophysical and cosmological bound systems, one finds a mismatch between the mass inferred by its gravitational effects and the mass inferred by other observables (electromagnetic ones), with the former much larger than the latter. The excess of the former with respect to the latter is dubbed DM.*

A couple of comments:

- For the moment there is no implication that DM is an exotic form of matter. It might still be ordinary matter which does not shine (e.g. dim stars, planets, cold and/or rarefied gas, etc.).
- The DM notion implicitly assumes that the theory of gravity used (Einstein GR, most often in its Newtonian limit, in fact!) is correct.
- The fact that it is denoted as "matter" (as opposed e.g. to radiation) has to do with the fact that its effects are inferred in bound systems, so that DM must "cluster" and form structures (this is very different, for instance, from "dark energy" that you will also touch upon in this school.)

Despite what you may have heard, you see that the DM hypothesis is born as a rather conservative explanation of observations. Let us sketch a few of the arguments leading to its (purely gravitational, then like now!) discovery, which goes back to the 30's of the XX$^{\rm th}$ century. For this, it is worth introducing some basic notions.

A. Vlasov and Jeans equations

The (one particle) phase space distribution of a system of particles only subject to an "average" gravitational force associated to the potential (per unit mass [1]) Φ obeys to

$$\left[\frac{\partial}{\partial t}+\vec{v}\cdot\frac{\partial}{\partial\vec{x}}-\frac{\partial\Phi}{\partial\vec{x}}\cdot\frac{\partial}{\partial\vec{v}}\right]f=0\,. \tag{1}$$

If Φ, rather than being externally assigned, is generated by the particles themselves (*Vlasov equation*), the above equation still constitutes a non-trivial problem, which must be solved in association to the Poisson equation (in the Newtonian limit):

$$\nabla^2\Phi=4\pi G_N n\equiv 4\pi G_N\int d^3\vec{v}\,f\,. \tag{2}$$

where we introduced the spatial density function $n(\vec{x})$ as the integral of the distribution function f over velocity. A more manageable set of equations only in spatial coordinates can be obtained with the method of moments, leading for the zeroth and first moment of velocity to the so-called *Jeans Equations* (summation over repeated index implicit)

$$\frac{\partial n}{\partial t}+\frac{\partial(n\bar{v}_i)}{\partial x_i}=0 \tag{3}$$

$$n\frac{\partial\bar{v}_j}{\partial t}+n\,\bar{v}_i\frac{\partial\bar{v}_j}{\partial x_i}=-n\,\frac{\partial\Phi}{\partial x_j}-\frac{\partial(n\,\sigma_{ij}^2)}{\partial x_i}\,. \tag{4}$$

Above, we introduced the expectation value of the velocity dispersion squared $(v_i-\bar{v}_i)(v_j-\bar{v}_j)$

$$\sigma_{ij}^2\equiv\overline{v_iv_j}-\bar{v}_i\bar{v}_j\,. \tag{5}$$

Proof: The first equation follows trivially from integrating Eq. (1) over the velocity space and the divergence theorem on f, noting that this function goes to zero for large $\vec{v}$. The second equation follows from multiplying Eq. (1) times v_j and the divergence theorem on f, then integrating. The final form is obtained by subtracting from the equation thus obtained the first equation times $\bar{v}_j$. The explicit derivation is left as an exercise, or see [1], Sec. 4.8.

Some comments:

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[1] We shall implicitly make throughout the hypothesis that DM constituents all share the same mass.

- Analogous equations are obtained in fluidodynamics, and are known as *(continuity and) Euler Equations:*

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \tag{6}$$
$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho}\nabla P - \nabla \Phi \,. \tag{7}$$

  where we introduced the fluid density ρ, and the pressure P replaces the σ^2_{ij}, assuming only isotropic stress present. This is one reason why one sometimes speaks of DM "fluid", and uses other similar notions by analogy. Be aware of the limitations, though!

- Neither the Jeans equations nor the Euler ones are closed equations. They are just the first two moments of a hierarchy which is in principle infinite (i.e. one would need a third equation for the six independent quantities σ^2_{ij}, in turn dependent on the third moments of velocity, etc.) However, in some approximations of particular relevance (e.g. some symmetry assumptions, a specified equation of state...) a closure is possible.

- Eq. (1) is itself an approximation, notably:

  a) It only describes the one-point distribution function, not addressing higher-order correlations between particles, including so-called collisional/two-body/relaxation effects in gravitational interactions, i.e. the fact that the gravitational interactions among DM particles are "granular", not strictly-speaking mean-field ones (for those of you who are familiar with it, it is only the first equation of the *BBGKY hierarchy*).
     This is usually an excellent approximation, due to the very large number of DM "particles" involved, but one exception of some importance is the case of stellar mass or heavier Black Holes (for a recent popular science review of this newly popular idea, see [2].)

  b) It is a classical (as opposed to quantum) equation.
     This is also ok for almost all DM candidates, one exception being e.g. "fuzzy" DM candidates whose de Broglie wavelength is of astrophysical scale, an idea recently becoming again popular following [3].

  c) It also ignores short-range interactions of non-gravitational nature, which are in general responsible for source/sink terms at the RHS. This may be suitable to describe DM system, but is certainly inadequate to deal with DM production, or techniques for its non-gravitational detection (more on this later).

1. *Application I: Oort's method to infer local matter density*

Let us assume a steady state solution of Jeans equations, $\partial n/\partial t = 0$. Using the resulting Eq. (3) in Eq. (4) leads to

$$\frac{\partial (n\,\overline{v_i v_j})}{\partial x_i} = -n\,\frac{\partial \Phi}{\partial x_j}\,. \tag{8}$$

To a first approximation the Galaxy (as seen from the Sun) can be modeled as as disk, homogeneous in the $x-y$ direction and much more extended radially than vertically. In this limit, the dominant gradients in the gravitational potential are vertical (hence derivatives with respect to z), and the only component of the Eq. (8) which matters is the z component, leading to

$$\frac{1}{n}\frac{d(n\,\overline{v_z^2})}{d\,z} = -\frac{d\Phi}{d\,z}\,. \tag{9}$$

This equation implies that the gradient of the gravitational potential can be deduced provided that one has access to two ingredients: i) A *tracer* of the density as a function of height above the plane, like for example a class of bright stars. It is important to note that it only matters that their number density $n_{\rm tr} \propto n$, since Eq. (9) is left unchanged by a $z-$independent rescaling of n. ii) A measurement of the vertical velocity dispersion of this tracers, $\overline{v_z^2}$, always as a function of z. On the other hand, the variation in the gradient with z allows one to infer the density of gravitating mass via the Poisson Equation,

$$\frac{d^2\Phi}{d\,z^2} = 4\pi G_N\, n\,. \tag{10}$$

Back in the thirties, Oort used K giant stars as tracers to develop such a program [4]. Recently, such methods infer (modern notation/values) a total *gravitational* density around the solar system of the order of $10 \times 10^{-21}\,{\rm kg\,m^{-3}}$, as opposed to the estimated value from "star counts" of the order of $4 \times 10^{-21}\,{\rm kg\,m^{-3}}$. Actually the majority (or at least a fraction similar to the visible one) of the mass in our neighborhood might be in some *unseen* form.

2. *Application II: Spherical systems, applications to Galaxy Clusters*

What is usually reported as the first "awareness" of the DM problem came from clusters of galaxies. In particular, F. Zwicky applied the virial theorem (obeyed by particles bounded in a system by conservative forces), linking average kinetic and gravitational potential energy $2\langle T\rangle = -\langle U\rangle$, to the Coma cluster [5, 6]. consider either one Galaxy of mass m orbiting a in a spherically symmetric system, at a distance r from a the center of the mass distribution, whose cumulative enclosed mass is $M(r)$. One has (using ergodic hypothesis)

$$2\frac{m\sum\overline{v_i^2}}{2} = r^{-1}G_N M(r)m\,, \tag{11}$$

hence from an estimate of the velocity dispersion (from doppler shifts in galactic spectra) as a function of distance r (inferred e.g. with geometric methods plus astronomical distance ladder) one can estimate the total mass $M(r)$, to be compared e.g. with photometric estimates. The former was (and still is) systematically larger than the latter.

Let us approach the problem from the point of view of Vlasov-Jeans equation. Galaxy clusters are generically fairly round and slowly-rotating. It is useful to derive the analogous "Jeans equation" (do it as a exercise, following e.g. [1], 4.210 and following) assuming spherical symmetry and stationarity, one obtains

$$\frac{d(n\,\overline{v_r^2})}{d\,r} + 2\frac{\beta}{r}n\overline{v_r^2} = -n\,\frac{d\Phi}{d\,r}\,. \tag{12}$$

where one also writes

$$\frac{d\Phi}{d\,r} = \frac{G_N M(r)}{r^2} \tag{13}$$

in terms of the $M(r)$, total mass enclosed within a distance r and we have introduced the velocity anisotropy parameter

$$\beta(r) = 1 - \frac{\overline{v_\theta^2} + \overline{v_\phi^2}}{2\overline{v_r^2}}\,. \tag{14}$$

One usually rewrites Eqs. (12,13) in the form

$$M(r) = -\frac{r\,\overline{v_r^2}}{G_N}\left[\frac{d\log n}{d\log r} + \frac{d\log\overline{v_r^2}}{d\log r} + 2\beta(r)\right]\,. \tag{15}$$

We immediately see what is the problem to infer a mass (profile) from the above equation: while radial profiles of some tracers (galaxies) can be obtained relatively easily, one needs *two* functions of the velocity distribution, while only the velocity distribution along the line of sight is accessible (spectroscopic determination of motion via Doppler shift). In modern times, this problem is usually addressed in clusters of galaxies by using the baryonic gas "as tracer" of the underlying gravitating mass: *If we assume* hydrostatic equilibrium in the gas, it obeys

$$\frac{dP_{\rm gas}}{dr} = -\frac{G_N\,M(r)\,\rho_{\rm gas}}{r^2} \tag{16}$$

by further assuming the thermal hypothesis, using $P = k_B\,T\,N/V$ with N the number of particles in the gas, which is written for a ensemble of average mass μ in units of atomic mass units m_u, $N = m/(\mu\,m_u)$, one derives $P_{\rm gas} = (\mu\,m_u)^{-1}k_B\,T_{\rm gas}\,\rho_{\rm gas}$ and

$$M(r) = -\frac{k_B\,r\,T_{\rm gas}}{\mu\,m_u\,G_N}\left[\frac{d\log\varrho_{\rm gas}}{d\log r} + \frac{d\log T_{\rm gas}}{d\log r}\right]\,. \tag{17}$$

From X-ray maps, one can deduce the enclosed mass via the gas density profile (correlated with intensity of the emission) and the temperature profile (related to the energy of measured X- rays), for an actual example see e.g. [7].

Compare Eq. (17) with Eq. (15): they would be formally identical in the case of velocity isotropy (i.e. $\beta = 0$) and "thermal" assumption for the velocity distribution $\overline{v^2} \propto T$. If these hypotheses are often (not always!) justified for the baryonic gas which is subject to "collisions", generically they are not true for DM.

3. Application III (sketch): Galactic rotation curves

Already in the 30's (see for instance [8]) it was noted that a galaxy like Andromeda presented an *unexpectedly large* circular velocity at large distances from its center. It was only by ~ 1970, however—also thanks to a number of technical improvements, such as radioastronomy, 21 cm tracer, improved spectroscopic surveys—that some pioneers like V. Rubin and W. K. Ford Jr. embarked in a systematic campaign to obtain rotational curves of Spiral Galaxies up to their faint outer limits, eventually reporting a universal indication for a flattening of these curves, see e.g. [9]. We shall sketch very simply where the puzzle comes from: At large distances from the "center"($\equiv$where virtually all of its luminous matter is concentrated) of the galaxy, one observes

$$v^2_{\rm rot,obs} \simeq const. \tag{18}$$

as opposed to the expectation

$$v^2_{\rm rot,exp} \simeq \frac{G_N\, M(r)}{r} \propto \frac{1}{r}\,, \tag{19}$$

where $M(r)$ is the mass enclosed within r, and the last equality is expected if all the mass is concentrated at small distance from the center of the distribution ($r \simeq 0$), like (after all!) for the Solar System. This problem could be solved by invoking a sizable amount of gravitating matter, roughly distributed in a spherical halo extending well beyond the luminous core of the galaxies, with profile $\rho(r) \propto r^{-2}$, so that $M(r) \simeq \int dr 4\pi\, r^2 \rho(r) \propto r$. By the way, the need for something of the sort was also obtained independently, via the first numerical simulations in the 70's, addressing the problem of the stability of disk galaxies: in articles such as [10] it was argued that rotationally supported systems (like the observable stars in many spiral galaxies, including the Milky Way, seem to be) were unstable, and their disk appearance should be lost in a few dynamical scales in favour of elongated structures. A way out was identified in the possibility that large, roughly spherical halos mostly supported by random velocities, were extending way beyond the observed luminous region.

Evidence for DM at galactic scales is perhaps the most troublesome: on the plus sign, it is comforting that something similar to what happens at cluster scales (Mpc) and in cosmology (Gpc) is also found down to kpc scales (basically, the dwarf galaxy scale). Additionally, the details of the DM distribution in our Galaxy (see e.g. [11]) and nearby ones is important towards DM identification via direct or indirect techniques (see below). Finally, at "sociological level", they came in at an epoch where people were more willing to take these evidences seriously, and the first particle physics models which could "naturally" accommodate for such phenomena were being elaborated. It is undeniable that since the end of the 70's or early 80's the DM problem has grown in prominence and consideration. However, these determinations are still relatively shaky with respect to the ones at larger scales, being affected by a number of assumptions (e.g. *assumed* symmetries of the system, steady state configurations) and by important, highly non-linear processes involving the baryonic material (such as feedbacks via supernovae explosions, star-burst episodes accompanied by winds, central black holes activities, etc.). The galactic scale is also an arena where modifications of gravity (MOND and the like), at least at phenomenological level, seem to account for a number of observations much more easily than naive expectations from vanilla DM models. It is also possible that the phenomena involving DM at these scales might depend to some extent by some "non-minimal" properties of the constituents of DM (like their non-gravitational interaction, some peculiar velocity distribution, etc.). This is an extremely active field of research (see more on this below) although at the moment it remains unclear how to disentangle clearly such putative effects from more mundane (but difficult to model) non-linear baryonic effects.

B. More modern evidence

None of the above evidences convincingly argues in favour of some exotic species constituting the DM. After all, it is not an unusual situation in astrophysics not to have a complete "account" of what is known to exist: For instance, we do know that a large number of small bodies populate the outer solar system beyond Neptune (possibly including also planetary-sized objects) but we are very far from a complete census of them. Most of the stars in the universe are very dim and escape detection (actually, even in our own Galaxy!). A large chunk of the baryonic material in the universe (whose existence we infer from cosmological arguments) is still unidentified ("missing baryons"), possibly residing as hot and very rarefied gas around galactic halos, or in between them [12]. And so on.

The situation has however changed dramatically with some modern evidence for DM:

> *Cosmological evidence strongly suggests that the DM phenomenon: a) should be attributed to a gravitating species, rather than to a Modification of Gravity (MG). b) cannot be accounted for via the known particles and forces of the standard model of particle physics, requiring some new ingredient.*

These conclusions are heavily based on cosmological perturbation theory, applied to the Cosmic Microwave Background anisotropies (CMB) and clustering properties of the large scale structures (LSS). The quantitative tools to study those are introduced in dedicated lectures. Here I will just summarize a few facts and the overall logic for these conclusions:

- We observe $\sim 10^{-5}$ anisotropies in the CMB, as well as a peculiar angular power spectrum, with peaks and throughs. The CMB was "freed" at an epoch corresponding to $z \simeq 1100$, before which photons and baryons (mostly protons, via their respective couplings with electrons) were strongly coupled. These anisotropies reflect: i) the temperature fluctuations at those recombination times ii) the gravitational redshift (blueshift) experienced in climbing up (down) the potential wells at the recombination time (*early Sachs Wolfe effect*, ESW) iii) a Doppler effect due to the relative velocity. iv) the same effect as ii), but integrated along the line-of-sight during the whole history of the universe/photon propagation, influenced by the following growth of structures (*integrated Sachs Wolfe effect*, ISW).

- Due to the e.m. coupling, the perturbations in the baryon density are linked to the ones in photons and of the same order at $z \simeq 1100$. If we were to run the Universe forward with those initial conditions (and no other component), the time between there and now would simply be insufficient for structures to grow to the observed level, under purely gravitational instability.

- It is conceivable that the level of structures quantified e.g. by the power-spectrum of density contrast of LSS, might still be attained by assuming a MG, with a stronger than standard growth of perturbations (this is at least qualitatively achieved in some so-called TeVeS theories [13] [2]). However, the baryonic fluid keeps memory of its former coupling to the photons, via peculiar oscillations in Fourier space ("acoustic" waves in the plasma). These are not due to gravity, but to the e.m. coupling, and one can hardly see how to "undo" them in MG. An account of this issue is given in [14]. Even the possibility that these features are missed due to observational problems and "smoothing" must be discarded, since we *do see* these features, at a "suppressed" level (since baryons only make a small fraction of the total matter), in the LSS (this are the so-called baryon acoustic oscillations or BAO).

- These problems disappear if we add a single ingredient: a DM "fluid" which gravitates but is not electromagnetically coupled to photons before recombination. Even if the initial perturbations created by some early universe mechanism were shared by photons, baryons and DM (as we believe it is the case to a high degree of approximation, in the so-called inflationary model with adiabatic initial fluctuations), by the time photons and baryons recombine, the DM density contrast has already grown somewhat: first logarithmically with the scale factor, in the radiation-dominated phase, then linearly in the matter dominated phase. The baryons can thus fall within the already significant DM potential wells soon after the photons recombine, and structures can form sufficiently early to agree with observations.

- A quantitative determination of how much DM is needed with respect to baryons is remarkably consistent across observables (CMB, LSS, Clusters). For instance, at least at the 10% level, the $\Omega_b/\Omega_{\rm DM}$ ratio inferred from CMB agrees with the one inferred from clusters of galaxies, suggesting that we are observing the same phenomenon at different scales. Additionally, since the CMB parameters are inferred in a *linear perturbation* phase, when departures from homogeneity and isotropy are only tiny, this indicates that the DM cannot be attributed to unidentified, but otherwise "ordinary" baryonic matter. In fact, Ω_b inferred from CMB is even consistent with the Ω_b deduced from primordial nucleosynthesis considerations, going back to a much earlier period.

Another important piece of evidence has been obtained in the last decades via gravitational lensing: on the one hand, detailed analyses of lensing due to clusters on background galaxy images requires "gravitating mass" in-between the galaxies of the clusters (and more smoothly distributed than them), dominating the total potential. On the other hand, there are several cases where colliding clusters of galaxies (bullet clusters, train-wreck, etc.) show segregation effects, with the cluster gas (which dominates the total baryonic gas of clusters, as opposed to galaxies where instead baryons in stellar form dominate) forming shocked fronts detected via X-rays, while lensing shows the gravitating mass passing through the shock front unscathed, just like galaxies (and collisionless species) do (for a review, see e.g. [16]). Actually, recently even secondary effects on the CMB due to lensing of the gravitational structures crossed have provided another independent cross-check of the DM scenario.

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[2] Note that this *does require* extra degrees of freedom. DM would thus be interpreted in MG as a manifestation of one or more "classical" fields, as opposed to "particles", i.e. excitations of a quantum field. But the need for extra ingredients in addition to the Standard Model of Physics would be the same, with the difference that most of the extended gravity sectors that might still make sense classically are known to be flawed at the quantum level. This problem is absent in any presently viable DM theory. Also, some authors intend DM exclusively as particles having other interactions in addition to gravity, but this is an unnecessary requirement. Be aware, thus, that to some extent the difference between modifying gravity with extra fields or adding extra matter fields with purely gravitational couplings is merely in semantics, see e.g. [15].

II. INTERLUDE - NOTIONS OF THERMODYNAMICS IN THE EARLY UNIVERSE

Exercise: Both in particle and astroparticle physics, natural units (where $c = k_B = \hbar = 1$) are used (on the other hand, differently from some convention frequent among people working on gravity, we retain $G_N \equiv M_P^{-2} = (1.22\times 10^{19}\,\text{GeV})^{-2}$). If you are unfamiliar with them, practice a bit!

- Compute your typical body temperature (assuming you are still alive) in eV.
- Check the working frequency of your mobile phone. Rephrase it into eV.
- Compute your height in eV^{-1}.
- Compute your age in eV^{-1}.
- Compute your density (estimate with $\mathcal{O}(10\%)$ error from what happens when you jump into the sea, a lake or a swimming pool+ Archimedes' law) in eV^4.

The following is a recap of notions introduced by other lecturers. For details, see any standard reference, e.g. [17].

The *cosmological principle* justifies seeking cosmologically relevant solutions to the Einstein eqs. in terms of a homogeneous and isotropic metric (at least statistically speaking and at large scales). The key equations for the time evolution of scale factor a and energy density ρ in the smooth (and *3D spatially flat*) universe are the so-called *Friedmann equations*

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G_N\,\rho}{3}\,, \tag{20}$$

$$\dot\rho + 3(\rho + P) = 0\,, \tag{21}$$

which must be supplemented by an equation of state, typically of the form $P = P(\rho)$. Given the most relevant types of energy: matter ($P \simeq 0$), radiation ($P = \rho/3$), and cosmological constant ($P = -\rho$), the scale factor evolution writes as

$$\frac{1}{H_0^2}\left(\frac{\dot a}{a}\right)^2 = \Omega_\Lambda + \Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_{r,0}\left(\frac{a_0}{a}\right)^4\,, \tag{22}$$

where $H_0 \equiv h\,100$ km/s/Mpc is the current value of the Hubble expansion constant ($h \simeq 0.7$), and the $\Omega_{i,0}$ are the energy density contents in different species normalized to the critical one today, $\Omega_{i,0} \equiv \rho_{i,0}/\rho_c$, $\rho_c \equiv 3H_0^2/(8\pi G_N)$.

Exercise: Compute the value of ρ_c in SI as well as natural units, and of H_0 in natural units.

The expansion of the universe explains "Hubble's" universal recession in terms of decreasing density of "matter particles" according to $n \propto a^{-3}$, or equivalently conservation of the covariant density $n\,a^3 = const.$, while radiation undergoes a further dilution due to the stretching of its wavelength. This phenomenon is called cosmological redshift (z), and

$$1 + z = \frac{\lambda_{\rm obs}}{\lambda_{\rm emiss}} = \frac{a_0}{a_{\rm emiss}}\,. \tag{23}$$

The cosmological principle also implies that, at least in the "cosmic" frame, the phase space distribution of particles only depends on $f = f(p,t)$, with $p = |\mathbf{p}|$. This is linked to useful quantities we shall need via the following (g is the particle multiplicity, e.g. 2 for a photon, 2 for a neutrino species, 4 for electrons):

- Current density of particles n^μ, whose only non-vanishing term is $n = n^0$ and only depends on t

$$n^\mu = g\int f\frac{p^\mu}{p^0}\frac{d\vec p}{(2\pi)^3} \Rightarrow n = \int f\frac{d\vec p}{(2\pi)^3}\,. \tag{24}$$

  Using the covariant derivative ∇_μ, current conservation writes

$$\nabla_\mu n^\mu = a^{-3}\frac{d(a^3 n)}{dt} = \frac{dn}{dt} + 3H\,n = 0\,, \tag{25}$$

  which is the formal expression of the above-mentioned condition of particle number conservation $n\,a^3 = const.$

- The stress energy tensor (which is what enters Einstein equations!) is defined as

$$T^{\mu\nu} = g\int f\frac{p^\mu p^\nu}{p^0}\frac{d\vec p}{(2\pi)^3}\,, \tag{26}$$

from which one derives the expression of the energy density

$$\rho = T^{00} = g\int f\,p^0\frac{d\vec{p}}{(2\pi)^3}\,, \tag{27}$$

as well as—taking into account spatial isotropy—of the pressure P

$$-P\delta^{ij} = T^{ij} = -\delta^{ij} g\int f\frac{|\vec{p}|^2}{3}\frac{d\vec{p}}{(2\pi)^3}\,. \tag{28}$$

Again, the 2nd Friedmann eq. can be obtained as the only non-trivial Bianchi identity or "energy conservation":

$$\nabla_\mu T^{\mu\nu} = 0 \Rightarrow \frac{d(a^3\rho)}{dt} = -3H(\rho+P)\,. \tag{29}$$

- Similarly to other thermodynamical observables, one can define an entropy density and current. At the classical level, this writes

$$s^\mu = -g\int f(\ln f - 1)\frac{p^\mu}{p^0}\frac{d\vec{p}}{(2\pi)^3} \;\Rightarrow s^0 = -g\int f(\ln f - 1)\frac{d\vec{p}}{(2\pi)^3}\,. \tag{30}$$

  When including quantum statistics effects, this generalizes to

$$s^\mu = -g\int [f\ln f \mp (1\pm f)\ln(1\pm f)]\frac{p^\mu}{p^0}\frac{d\vec{p}}{(2\pi)^3}\,, \tag{31}$$

  with the upper (lower) sign for bosons (fermions) and which clearly reduces to Eq. (30) for $f \ll 1$, since $\mp(1 \pm f)\ln(1\pm f) = -f + O(f^2)$.

Provided that the interaction rates of the processes keeping a given species in thermal contact with the bath are fast enough (compared to the Universe expansion), at least locally the Universe attains thermodynamical equilibrium (Instead, why local thermodynamical equilibrium quantities seem to be shared at a "global" level is one of the puzzles suggesting inflation!). The phase-space distribution then is well known from basic statistical theory,

$$f(p) = \frac{1}{\exp\left(\frac{E-\mu}{T}\right)\pm 1}\,, \tag{32}$$

where the upper $(+)$ sign refers to fermions, the lower $(-)$ to bosons.Temperature T is the intensive quantity (or a "multiplier") associated to the exchange of energy with the bath, μ the analogous one related to exchange of particles. If processes leading to *energy* exchange are "fast", one says that *kinetic* equilibrium is attained. If processes leading to *particle* exchange are "fast", one says that *chemical* equilibrium is attained. *Thermal* equilibrium usually is meant to imply *both*. At equilibrium, the above-introduced quantities attain a simple expression in terms of T (we set $\mu = 0$ in the following):

- Number density, relativistic limit

$$n = g\int f\frac{d\vec{p}}{(2\pi)^3} = \frac{g}{2\pi^2}T^3\mathcal{J}_\pm(2) \tag{33}$$

- Energy density, relativistic limit

$$\rho = g\int Ef\frac{d\vec{p}}{(2\pi)^3} = \frac{g}{2\pi^2}T^4\mathcal{J}_\pm(3) \tag{34}$$

- Pressure, relativistic limit

$$P = \frac{1}{3}\int\frac{p^2}{E}f\frac{d\vec{p}}{(2\pi)^3} = \frac{\rho}{3}\,. \tag{35}$$

  In the above equations we defined (remember that for integer a, $\Gamma(1+a) = a!$)

$$\mathcal{J}_\pm(a) \equiv \int_0^\infty dy\frac{y^a}{e^y\pm 1} = \begin{cases}\Gamma(1+a)\zeta(1+a) & -\,,\ \text{bosons}\,,\\ (1-2^{-a})\Gamma(1+a)\zeta(1+a) & +\,,\ \text{fermions}\,.\end{cases} \tag{36}$$

Note that applying comoving particle conservation to relativistic species, one derives $n\,a^3 = const.$, hence $T\,a = const.$, which allows one to use the temperature of a thermal relativistic species (typically photons) as a "clock", replacing the scale factor or time as independent variable.

In the non-relativistic limit ($T \ll m$, $E \simeq m + p^2/(2\,m)$), the thermodynamical quantities are the same for bosons and fermions and one has the Maxwell-Boltzmann distribution, $f_{\rm MB}(p) = \exp(-p^2/(2\,m\,T))\exp(-m/T)$. From that follows

- Number density, non-relativistic limit

$$n = g\int f\frac{d\vec{p}}{(2\pi)^3} = g\left(\frac{m\,T}{2\pi}\right)^{3/2}\exp\left(-\frac{m}{T}\right) \tag{37}$$

- Energy density, non-relativistic limit (remember $E = m + p^2/(2\,m)$)

$$\rho = g\int Ef\frac{d\vec{p}}{(2\pi)^3} = n\left(m + \frac{3}{2}T\right)\,. \tag{38}$$

- Pressure, non-relativistic limit

$$P = \frac{1}{3}\int\left[\frac{p^2}{m} + \mathcal{O}\left(\frac{p^2}{m^2}\right)\right]f\frac{d\vec{p}}{(2\pi)^3} = n\,T\,. \tag{39}$$

For the entropy density, it is useful to keep in mind the following relation for the species in equilibrium

$$s = \frac{\rho + P - \mu\,n}{T}\,. \tag{40}$$

It is sometimes useful to rewrite the total density in the relativistic period as

$$\rho_{\rm tot} = \frac{\pi^2}{30}g_{\rm eff}(T)T^4 \tag{41}$$

or equivalently (via first Friedmann Eq.)

$$H^2 = \frac{4\pi^3}{45\,M_P^2}g_{\rm eff}(T)T^4\,, \tag{42}$$

where

$$g_{\rm eff}(T) \simeq \sum_{\rm i=relat.bos.} g_i\left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{\rm j=relat.ferm.} g_j\left(\frac{T_j}{T}\right)^4 \tag{43}$$

counts the degrees of freedom (only the relativistic ones, the non-relativistic ones contributing negligibly in the relativistic era) and accounting for the different statistics as well as for the possibility that different species attain different temperatures. This is actually the case, to a good approximation, for neutrinos and photons at temperatures well below the electron mass, with $(T_\nu/T_\gamma)^3 \simeq 4/11$, while being the same for $T \gg m_e$.

Exercise: Derive this result via entropy conservation in the photon fluid before and after $e^\pm$ annihilation, while considering ν's coupled to photons (and thus sharing their T) at $T \gg m_e$ and decoupled by the time $e^\pm$ annihilations take place.

Also note that the entropy density is always dominated by the relativistic species, $s \simeq (4/3)\rho/T$ and in absence of particle creations/annihilations, it is conserved! It is useful to write it as

$$s = \frac{2\pi^2}{45}h_{\rm eff}(T)T^3\,, \tag{44}$$

where

$$h_{\rm eff}(T) \simeq \sum_{\rm i=relat.bos.} g_i\left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{\rm j=relat.ferm.} g_j\left(\frac{T_j}{T}\right)^3\,. \tag{45}$$

The quantity s is used (interchangeably with n_γ or T^3, more or less) to normalize densities to a comoving quantity.

Exercise: Check explicitly the above integrals.

III. DM PROPERTIES AND BASICS ON MODELS AND PRODUCTION MECHANISMS

A. Cosmologically inferred properties

Cosmological and astrophysical observations do not only tell us about the existence of the DM phenomenon, also tell us something about DM properties. Here is a(n incomplete) list:

1. DM abundance

Data tell us *how much* DM is out there, as summarized via the parameter $\Omega_{\rm DM}$. Other lectures tell you more about how this parameter is measured. Later on, we shall dwell on how it can be computed, in some given models.

2. DM velocity distribution

DM must have a *non-relativistic* velocity distribution, so it cannot be "hot" as the SM neutrinos are (which is one of the reasons SM neutrinos cannot make the DM, besides not having a sufficiently large mass).This has essentially to do with the properties of large-scale structures that we observe: a too fast DM candidate would not settle into potential wells below a characteristic scale, known as *free-streaming scale*. For a given DM candidate, one can estimate this scale (no proof given here, but should sound reasonable to you!)

$$\lambda_{fs}(t) = a(t)\lambda_{fs}^{\rm com} \simeq a(t)\int_{t_{\rm kd}}^{t} dt' \frac{\overline{v}_{\rm kd}}{a(t')^2}\,. \tag{46}$$

where $\overline{v}_{\rm kd}$ is a typical velocity the particle has at the time it decouples from the plasma, $t_{\rm kd}$. One clearly has to ask it to be at most as large as kpc, where DM structures have been detected.

3. DM lifetime

DM must be remarkably long-lived for particle physics standards, since it has to survive at least for the age of the universe (actually, at least one order of magnitude longer, see e.g. [18, 19] and possibly much more, if its decay products contain "visible" SM byproducts). Apart from some comments below, we will not indulge too much on this, but be aware that it is an extremely important constraint for model-building, see e.g. [20].

4. DM mass

DM is detected in virialized structures as small as dwarf galaxies, characterized by kpc-size and velocity dispersions of O(10) km/s or less. Its behaviour appears correctly described by classical mechanics at and above those scales. This suggests that associated quantum phenomena must be shorter scales, e.g. that its De Broglie wavelength satisfies

$$\lambda_{DB} = \frac{h}{m\,v} \lesssim 1\,{\rm kpc} \Rightarrow m \gtrsim 10^{-21}\,{\rm eV}\left(\frac{\lambda_{DB}}{1\,{\rm kpc}}\right)\left(\frac{10\,{\rm km/s}}{v}\right)\,. \tag{47}$$

If DM is fermionic, a much stronger bound applies: i) DM obeys Fermi-Dirac statistics ("Pauli principle"), so that its initial phase-space density has the upper limit (g being its multiplicity)

$$f \leq \frac{g}{h^3}\,. \tag{48}$$

Additonally, under purely gravitational interaction the phase space density is preserved (Liouville theorem) and it can also be shown that for any observable the coarse grained phase space density must be lower than the real one, so that the above relation allows to derive a bound from current observations. This argument, initially developped in [21], currently allows to put a lower limit to fermionic DM of about 0.4 keV, see [22].

Exercise: Consider a spherical, uniform system, gravitationally bound, of mass M and radius R, made of cold fermionic particles of mass m, with spin multiplicity g. Compute the lower limit on m. Some hints/guidelines: *gravitationally bound* implies that all particles should have a velocity lower than the gravitational escape velocity (compute the latter for such a system!) *cold fermions*: the system contains $N = M/m$ particles. But the "coldest" fermion system still settles in "Fermi levels": compute the Fermi velocity as a function of M,R, m, g. *estimate:* Given the dependence on M, R, which systems are optimal to set bounds on m? Search the literature for typical values of M, R of astrophysical systems (clusters of Galaxies, Milky Way-like Galaxy, dwarf spheroidals) and to derive a(n order of magnitude) bound. *Bonus:* Direct (terrestrial) experiments tell us that neutrinos have a maximum allowed mass of about 2.2 eV. What is the maximum overall mass they could contribute to a Milky Way-sized halo (assuming that they can be "maximally packed" into it)? What if the forthcoming Katrin experiment pushes the upper limit to 0.2 eV, as expected?

5. DM self-interactions: elastic collisions

DM must be collisionless, at least if compared with typical collisional rates of baryonic gases. Too important collisions would spoil e.g. the segregated DM distribution inferred from colliding clusters like the bullet cluster, or modify a number of other observables, as reviewed in [23]. Typical bounds thus obtained are $\sigma/m \lesssim 0.1 - 1\,\mathrm{cm}^2/\mathrm{g}$, with some dependence on the velocity (in the simplest cases, some sort of "hard-sphere" scattering limit is assumed, although interactions mediated by massive particles are also considered in some cases). For comparison $1\mathrm{cm}^2/\mathrm{g}$=1.78 b/GeV, so that these bounds are quite loose from the particle physics (as opposed to atomic physics) point of view. Note that a self-interaction near these bounds may also be beneficial for some phenomenological consequences, so that models predicting such relatively large cross-sections are currently pursued, as well as possible specific signatures, see e.g. [24].

Exercise: i) Why are the above bounds on σ/m, rather than on σ? (*hint*: The physical quantity constrained is the interaction rate. How does it write in terms of fundamental quantities?) ii) Estimate the *geometric* cross section for hydrogen atomic scattering, and compare it with the above bounds.

6. DM self-interactions: inelastic collisions

DM should be dissipationless, i.e. to a large extent it cannot dissipate its energy by emitting some sort of radiation (including "dark" one). Otherwise, DM would behave similarly to baryons, for instance forming flat disks rather than spheroidal halos. This is relevant if DM interactions are mediated by relatively light particles, which are kinematically accessible in present-day halos, despite the relatively low DM velocities. Please do not get confused by articles like [25], which may superficially make you think that DM can be dissipative: they in fact only refer to *at most* a small fraction of DM, quantified e.g. in less than 5% in [26], see also [27].

Exercise: Assume the DM particle has a mass of 1 TeV, and that the DM velocity distribution in a halo is Maxwellian. Estimate below which mass of the "dark radiation/mediator" particle its on-shell production is possible in half of the DM-DM collisions in: i) the Milky Way Galaxy ii) The Coma cluster of galaxies.

7. DM interactions with SM.

DM cannot obviously interact too much with SM particles. . . otherwise it would not be "dark"! For most popular DM models direct, indirect or collider searches provide the most stringent bounds on these couplings (see below). But a number of bounds have been derived from astro-cosmo observables also on interaction with SM. Definitively, cosmological bounds on the DM interaction rate with photons [28] and neutrinos [29] are the most stringent ones: If too large, they would damp power-spectrum features at small scales. Considerations from astrophysics and cosmology (examples are related to the stability of structures such as galactic disks) also exist on interactions with the nucleons, see e.g. [30] or [31] for macroscopic DM candidates.

B. Predicting DM abundance - WIMPs

How to produce DM? To compute that, we know for sure that we need to go beyond Eq. (1), since the initial condition $f = 0$ (no DM to start with) is a solution of that equation. We shall not develop an advanced theory for that, but will just proceed heuristically, writing down the simplest equation for the number density of particles (i.e. integral of f over velocities) which provides a working DM production mechanism. This formalism basically goes back to [32]. We need to extend the SM with a (meta)stable particle X, directly (or indirectly, but let's neglect that here) coupled to the plasma of SM particles populating the early universe. In symbols, for a self-conjugated particle we have $XX \leftrightarrow (\mathrm{SM\,thermal\,bath})$. Let us imagine that the particle is in thermal (kinetic and chemical) equilibrium. We also know it should be non-relativistic, at the epochs we care about. If m_X is its mass, we only need to know its number density at the epochs of interest, since $\Omega_X \propto m_X n_X$. In particular,

$$\Omega_X\, h^2 = \frac{\rho_X\, h^2}{\rho_c} = \frac{m_X\, n_X\, h^2}{\frac{3H^2}{8\pi G_N}} = \frac{m_X\, s_0\, Y_0}{1.054\times 10^4\,\mathrm{eV\,cm^{-3}}} = 0.274\frac{m_X}{\mathrm{eV}}Y_0\,, \tag{49}$$

where we used ($h_{\rm eff}\simeq 2+3\times 2(4/11)7/8\simeq 3.91$ comes from accounting for γ's and ν's)

$$s_0 = 2889\left(\frac{T_{\gamma,0}}{2.725}\right)^3\,\mathrm{cm^{-3}}\,. \tag{50}$$

The abundance of the species (assumed non-relativistic) at thermal equilibrium is given by the Boltzmann distribution,

$$n_{X,\rm eq} = g\left(\frac{m_X T}{2\pi}\right)^{3/2}\exp\left(-\frac{m_X}{T}\right)\,, \tag{51}$$

where we implicitly assume that the rate Γ of interactions keeping the equilibrium are fast with respect to the Hubble expansion H, and that on much longer timescales than the interaction ones, the temperature adiabatically responds to the universe expansion, $T=T(a)$. In the opposite limit $\Gamma\ll H$, the number density of particles is frozen-out and particles are simply comovingly conserved (see sec. II), with $n_X\propto a^{-3}$. If the reactions keeping X in thermal contact are $2\leftrightarrow 2$, an equation with the correct (limiting) behaviour is

$$\frac{dn_X}{dt} + 3H\,n_X = -\langle\sigma v\rangle[n_X^2 - n_{X,\rm eq}^2] \tag{52}$$

with $\Gamma\propto\langle\sigma v\rangle$ (to be specified below). Let us simplify a bit this equation by rewriting it in terms of:
i) the comoving quantity $Y\equiv n_X/s$ under the assumption that the entropy is conserved, so that

$$\frac{dY}{dt} = -\langle\sigma v\rangle\, s\,[Y^2 - Y_{\rm eq}^2]\,. \tag{53}$$

Exercise: Prove the above.
ii) by introducing the dimensionless independent variable $x\equiv m/T$ (any mass scale is fine, although often one takes $m=m_X$)

$$\frac{dY}{dx} = -\frac{\langle\sigma v\rangle(x)\, s(x)}{x\,H(x)}\,[Y^2 - Y_{\rm eq}^2]\,. \tag{54}$$

Exercise: Prove the above.
Exercise: Formulae which can be applied also in more general cases, e.g. with entropy non-conservation:

$$\frac{dY}{dx} = -\sqrt{45\pi}M_P\, m\frac{h_{\rm eff}(x)\langle\sigma v\rangle}{\sqrt{g_{\rm eff}(x)}x}\left(1-\frac{1}{3}\frac{d\log h_{\rm eff}}{d\log x}\right)[Y^2 - Y_{\rm eq}^2]\,, \tag{55}$$

can be found in [33, 34]. Check them out.
We can further re-write

$$\frac{x}{Y_{\rm eq}}\frac{dY}{dx} = -\frac{\Gamma_{\rm eq}}{H}\left[\left(\frac{Y}{Y_{\rm eq}}\right)^2 - 1\right]\,. \tag{56}$$

with $\Gamma_{\rm eq}\equiv n_{\rm eq}\langle\sigma v\rangle$. In this form, the equation clearly shows the importance of the ratio $\Gamma_{\rm eq}/H$ in determining the outcome. Qualitatively, we expect that when (*if!*) $\Gamma_{\rm eq}/H\gg 1$ the abundance tracks equilibrium, $Y=Y_{\rm eq}$. When $\Gamma_{\rm eq}/H\ll 1$ (which at some point will happen, given the difference temperature dependence of the two) the RHS is vanishingly small, and Y is frozen (constant). An analytical estimate of this constant value can be obtained by searching for x_F such that

$$\Gamma_{\rm eq}(x_F) = H(x_F)\ \ \mathrm{freeze-out\ condition}\,. \tag{57}$$

If it turns out that freeze-out happens when the particle X is relativistic, the computation of Y is relatively simple, since Y is constant in this regime, and given by

$$Y = \frac{n_X}{s} = \frac{45\zeta(3)\,g}{2\pi^4}\frac{\{1(B),3/4(F)\}}{h_{\rm eff}(x_F)}\,,\Rightarrow \Omega_X\,h^2 = 0.0762\frac{m}{\mathrm{eV}}\frac{\{1(B),3/4(F)\}}{h_{\rm eff}(x_F)} \tag{58}$$

where x_F is given by Eq. (57), i.e.

$$x_F\sqrt{g_{\rm eff}(x_F)} = \langle\sigma v\rangle(x_F) m\, M_P\{1(B), 3/4(F)\}\frac{g\zeta(3)\sqrt{45}}{2\pi^{7/2}}\,. \tag{59}$$

For the case of neutrinos ($h_{\rm eff}=10.75$, $g=2,\ldots$), this gives $\Omega_\nu h^2 = \sum m_\nu/(94\,{\rm eV})$, which makes them unsuitable DM candidates (even forgetting about their "hotness"!) given the current upper limits on their mass.

If—as in practice required for a good DM candidate—the particle decouples when non-relativistic, Y varies in time. Although Eq. (56) cannot be integrated in closed form (it is a *Riccati* equation) we estimate the relic abundance as $Y_0 = Y_{\rm eq}(x_F)$, with

$$Y_{\rm eq}(x_F) = \frac{g}{h_{\rm eff}}\frac{45}{2\pi^2(2\pi)^{3/2}}x_F^{3/2}e^{-x_F}\,, \tag{60}$$

with the freeze-out condition Eq. (57) giving

$$x_F^{1/2}e^{-x_F} = \sqrt{\frac{4\pi^3\,g_{\rm eff}(x_F)}{45}}\frac{(2\pi)^{3/2}}{g\,M_P\,m\langle\sigma v\rangle(x_F)}\,, \tag{61}$$

implying

$$Y_{\rm eq}(x_F) = \sqrt{\frac{45\,g_{\rm eff}(x_F)}{\pi}}\frac{x_F}{h_{\rm eff}(x_F)M_P\,m\langle\sigma v\rangle(x_F)} \sim \mathcal{O}(1)\frac{x_F}{M_P\,m\langle\sigma v\rangle(x_F)}\,, \tag{62}$$

where x_F can be obtained from the Eq. (61), i.e. re-writing it as $x_F^{(i+1)} = G(\ldots, \ln x_F^{(i)})$ and starting from a first guess $x_F\sim 10$ (a more typical value for popular candidates is $x_F\sim 30$). Note

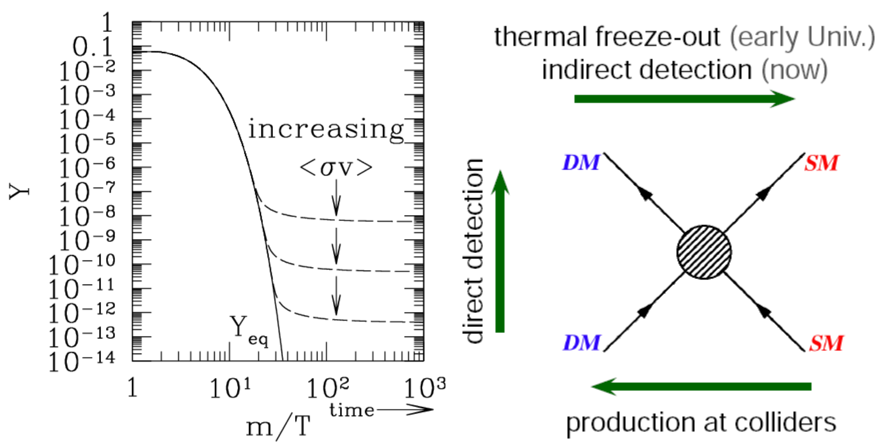

FIG. 1: Left: Schematics of WIMP decoupling. Right: Schematics of WIMP searches

- The more a particle interacts, the less of it there is: it makes sense, since the particle stays longer in thermal contact.
- a numerical estimates, assuming $\langle\sigma v\rangle \simeq const.$, leads to $\Omega_X\,h^2 = 0.1\,{\rm pb}/\langle\sigma v\rangle$. For electroweak scale masses and couplings (hence the name of *WIMPs, weakly interacting massive particles*) one gets the right value,

$$\langle\sigma v\rangle \simeq \frac{\alpha^2}{m^2} = 1\,{\rm pb}\left(\frac{200\,{\rm GeV}}{m}\right)^2\,. \tag{63}$$

  Since however the pre-factor depends from widely different cosmological parameters (Hubble parameter, CMB temperature) and the Planck scale, one might wonder if this is a mere coincidence or it hints to a more physical link. *This is usually dubbed WIMP miracle.*
- One may reverse the argument above, which is in fact what happened with WIMP models and largely explains the popularity of the DM problem among particle physicists. If one has a strong prior for new TeV scale physics (with ew.

strength coupling) due to the *hierarchy* problem (why is the Higgs light, compared e.g. to the Planck scale, given that its mass should receive quantum corrections proportional to new mass scales?), precision ew data (e.g. from LEP) suggest that tree-level couplings SM-SM-BSM should be avoided. A straightforward solution (not unique!) is to impose a discrete "parity" symmetry e.g.: R-parity in SUSY, K-parity in extra-dimensional models, T-parity in Little Higgs. New particles only appear in pairs! On the one hand, this has some additional phenomenological benefits (e.g. respecting proton stability bounds. . .), on the other hand, it automatically makes the lightest particle of this new sector stable! In a sense, some WIMP DM (albeit we do not know if too few, too many . . . or just the right amount) is "naturally" expected for consistency of the currently favored framework for BSM physics at EW scale. Beware of the reverse induction, though: LHC is current our best tool to test this paradigm, but if no new physics is found at EW scale it is at best the WIMP scenario to be disfavored, not the "existence of DM".

- The simple treatment presented above fails in some specific cases, such as cohannihilations with other particle(s) close in mass, or when the cross-sections strongly depend on energy, for instance in presence of resonances, thresholds, etc. For more details see for instance [35, 36]. Nowadays, relic density calculations have reached a certain degree of sophistication and are automatized with publicly available software, such as MicrOMEGAs (http://lapth.cnrs.fr/micromegas/) or DarkSUSY (http://www.physto.se/˜edsjo/darksusy/)

Exercise: Apply the freeze-out relic abundance formula to protons/antiprotons, assuming for the annihilation cross section $\langle\sigma v\rangle \simeq m_\pi^{-2}$, where m_π is the pion mass. Do you get any sizable abundance of protons produced this way, compared with the observed $\Omega_b \simeq 0.2\,\Omega_{\rm DM}$?

Exercise: By using any software of your choice (including symbolic ones like Mathematica©, Maple©, etc.), write a simple code solving the relic abundance equation. Compare with the analytical approximations discussed during the lecture. Feel free to explore what happens under different conditions (e.g. different dependences for the cross section; epochs of entropy variations. . .).Have a look e.g. at [37] for comparison and for some "tricks" on how to make the computation more efficient (notably if you find, as you probably should, problems of numerical stiffness).

1. *More on the annihilation cross-section*

In a collision between particle 1 and 2, one can define the average

$$\langle\sigma v\rangle \equiv \int {\rm d}^3\mathbf{v}_1 \int {\rm d}^3\mathbf{v}_2 h(\mathbf{v}_1)h(\mathbf{v}_2)\,\sigma(v_{\rm rel})v_{\rm rel}\,. \tag{64}$$

where the cross section only depends on the absolute value of the relative velocity. Remember that for any velocity distribution h one has

$$\int {\rm d}^3\mathbf{v}\, h(\mathbf{v}) = 1\,. \tag{65}$$

By exploiting this fact, the independence of the cross-section with respect to translations and changing variables, we can factorize our the movement of the CM with respect to the relative velocity, hence simplifying into

$$\langle\sigma v\rangle \equiv \int {\rm d}^3\mathbf{v}\, h(\mathbf{v})\,\sigma(v)v\,. \tag{66}$$

In a thermal-environment, $h(\mathbf{v}) \propto \exp[-v^2/(2\,T)]$. Away from particle-physics peculiarities (such as resonances, thresholds, etc.), keeping in mind that DM has to be non-relativistic, it makes sense to expand $\sigma(v)v$ into a power-series of velocities. In fact, partial wave expansion (see any standard textbook, including e.g. [38]) leads to a series in v^{2L}, L being the orbital angular momentum of the pair: in spectroscopic notation, $L = 0$ is the "S-wave" contribution, $L = 1$ the "P-wave", etc. In this approximation, the average over the termal distribution leads to $\langle\sigma v\rangle = a + bT + \ldots$ The first-principle calculation of the coefficients a, b, etc. goes beyond the level of these lectures. It is enough to say that $\langle\sigma v\rangle$ is proportional to the modulus square of the amplitude of the transition, $|\mathcal{M}|^2$, where the ampitute $\mathcal{M}$ can be computed with Feynman rules (if the theory is perturbative, that is).

C. Intro to alternatives

This is a non-exhaustive list, leaving out other often discussed candidates, such as sterile neutrinos, super-WIMPs, etc.

1. FIMP

We solved the evolution equation for Y under the assumption of initial equilibrium abundance, $Y(x \ll 1) = Y_{\rm eq}$, but this is unnecessary. Had we started with $Y(x_0 \ll 1) = 0$, provided that $\Gamma_{\rm eq}/H = K \gg 1$, Eq. (56) admits the solution $Y \simeq Y_{\rm eq} K \ln(x/x_0)$ [assuming K constant…which is not!] so equilibrium is attained when $x \sim x_0 \exp(1/K)$, i.e. only a 10% increase wrt x_0 for $K = 10$!

However, if $\Gamma_{\rm eq}/H = K \ll 1$ (i.e., *feeble* coupling!) the species never attains equilibrium: yet, X it can match the required DM value via the residual production from the plasma. That mechanism is called "freeze-in", since it's the reverse of the freeze-out (although it represents the early stage, of pre-thermalization, for each species eventually undergoing freeze-out). For more details see for instance [39]. Just note that, since $Y \ll Y_{\rm eq}$, we can neglect the former with respect to the latter at the RHS, obtaining

$$\frac{x}{Y_{\rm eq}} \frac{dY}{dx} \simeq \frac{\Gamma_{\rm eq}}{H} \,. \tag{67}$$

showing that the right abundance can be obtained if the integral of the RHS is smaller than unity, so that Y now freezes-in at values $x \simeq 1$. This leads to the opposite scaling $Y_\infty \propto \langle \sigma v \rangle$. More quantitative details can be found e.g. in [40].

2. SIMPs and friends

It is possible that DM is to large extent secluded from the SM. In principle, there might be a whole "dark sector", perhaps with new forces/mediators, which controls the properties of DM, including its relic abundance. One example is given by the "forbidden" DM, see [41]. Another recently popular alternative is that DM abundance is not set via particle-changing $2 \leftrightarrow 2$ processes with the SM bath, rather via $3 \to 2$ reactions purely in the DM sector. For this to be efficient, DM must be sufficiently strongly self-interacting (hence the name SIMP) via a coupling $\sqrt{4\pi\alpha_X}$ and quantitatively one expects their mass to be in the MeV to GeV range, see [42–44]. In particular, the RHS goes with the cube of the density, hence $\Gamma_{3\to 2} = \langle \sigma v \rangle_{32} n^2$ and $\langle \sigma v \rangle \simeq \alpha_X^3 / m_X^5$. Without further ingredients, the mechanism is doomed to fail, since the entropy transfer heats up the DM sector to unacceptable level. Hence *kinetic equilibrium* via $2 \leftrightarrow 2$ processes is needed for viable DM candidates. This might happen in a purely dark sector (e.g. dark photons), which reduces the possible signatures of the model to departures from the pure collisionless and dissipationless nature of DM. Or it might happen via (suppressed) coupling with the SM, which, if parameterized via $\langle \sigma v \rangle_{\rm kin} = \epsilon^2 / m_X^2$, require $10^{-9} \lesssim \epsilon \lesssim 10^{-6}$. This might or might not be challenging to achieve, but usually requires extra physics scales/ingredients, although there is hope that these values for the coupling may make the model amenable to tests (either at *high-intensity*, rather than *high-energy*, collider experiments, or via new technologies in direct detection techniques, suitable for lighter DM). Other models of strongly interacting DM arise in theories trying to link DM generation with baryon asymmetry generation, see e.g. [45, 46] which constitutes another avenue recently popular for model-building.

3. Beyond

DM may be produced via completely different mechanisms: An important class can be better understood by studying the evolution of a scalar field in a curved (FLRW) spacetime. We just mention here two examples of this class: i) Misalignment mechanism. ii) Gravitational production.

i) Imagine having a (pseudo)scalar field with a random initial value in the early universe. At a timescale set by its mass, the fields starts oscillations around the minimum of its potential (with amplitude given by the initial value). Provided that the damping (i.e. via decays) is negligible, the field energy density evolves as dark matter, i.e. as a^{-3}. In general, for this to work one requires light particles (sub-eV) and large values for the initial field displacement, so that (pseudo)Nambu-Goldstone bosons associated to the breaking of Global Symmetries provide a natural candidate (best known example is the *axion*, linked to a dynamical explanation of the *strong CP problem*).

ii) Generically, in time-dependent gravitational backgrounds, spontaneous particle creation takes place ("an initial vacuum state is no longer a vacuum state at later times", Bogolyubov transformations allow one to compute the number density of particles thus produced . . .). In most cases this phenomenon is not relevant for DM, but if massive particles exist with mass comparable with the expected Hubble rate at the end of Inflation, e.g. 10^{13} GeV, then they may account for DM [47, 48] and may even be related to features in UHECRs [49].

Note the broad difference of masses in the two cases, from μeV to ZeV! Overall, the hope is that the mystery of DM is solved via an ingredient useful to understand something else, rather than by a ad hoc solution, albeit the latter possibility should not be logically dismissed. More and more often, today, the attitude is: look for everything that can be looked for. This is a perfectly acceptable attitude, provided that one is ready to buy the consequence: there is no a priori reason to expect a detection. In particular the example (ii), in the limit where the DM lifetime is sufficiently long, provides a proof of principle that viable DM candidates exist which are virtually undetectable other than gravitationally. This is not a unique case, and the logical possibility that we *might never experimentally know* the fundamental properties of DM is a (perhaps disturbing, yet true) lesson to be kept in mind.

IV. SEARCHING FOR SIGNATURES (MOSTLY WIMP!)

One of the unique features of the WIMP scenario is that: i) they can be searched with many strategies; ii) the parameters probed can be compared across different types of searches. Although this has to be taken with a grain of salt in actual models, it has to do with the fact that the basic "toy diagram" (see right panel in Fig. 1) enters WIMP annihilations, scattering with the SM, or pair production in SM particle collisions. The fact that in many cases the detectors needed for these searches were already being developed for other purposes has also been a major plus in searching for this type of DM. But beware of the "street-lamp" effect: looking where we can because we can is not a gauge of success!

A. Collider searches

To explain DM, the SM should be extended by at least a new (neutral, uncolored) massive state, usually the lightest state of a new sector, otherwise made of unstable particles to which DM couples (and usually the SM, too). The most effective search strategy heavily depends on the spectrum (and couplings) of the new states. This makes searches at colliders intrinsically model-dependent. There are different approaches possible:

I *(Several) states heavier than DM are kinematically accessible*
In this case, the collider searches are mostly sensitive to those additional states (notably the coloured ones, like gluinos or stops in SUSY), which are directly constrained. Then, *theoretical* relations (depending on a somewhat meaningful UV model one is dealing with) are used to infer implications for DM, into which new states eventually decay into. This is for instance the "traditional" approach in supersymmetric DM candidate searches, see e.g. [50]. Since these model spaces are usually huge, one starts with some simplifying assumptions (e.g. assume all new fermions share the same mass scale, all new bosons as well, no extra CP violation, . . .) then pick "benchmark" models in different corners of the large parameter space, trying to be representative. While it is still a way that is pursued (and worth doing so!), especially in the light of negative results of BSM searches, it is more and more doubtful that the lessons learned this way are actually *generic*. Even if something like SUSY is realized at the EW scale, it appears less and less plausible that it is realized in some form close to the constrained MSSM.

II *No states heavier than DM kinematically accessible*
In this case, since the process $SM + SM \to XX$ is invisible, one has to rely on some visible particle against which the DM can recoil, thus leading to *(mono)jets/photons/massive gauge bosons* (unavoidably produced at least as initial state radiation)$+$ *missing energy* as signature. If one computes these observables in well-behaved UV theories as above, it turns out that these channels do not usually lead to the most stringent bounds. On the other hand, via these channels one may think of performing more *generic* searches, by integrating out the heavier states and construct an Effective Field Theory (or directly parameterize BSM as EFT, if agnostic on UV-theory), analogous to Fermi theory vs. Weak interactions. The additional advantage is that, since the same operators are leading to DM-SM scatterings, collider bounds can be compared to results of direct detection experiments (this exercise suggests that, at face value, LHC is better than direct detection at constraining relatively light DM, below $\sim$10 GeV). This strategy is however troublesome at LHC, since the typical exchanged-momenta in DM producing processes is not negligible with respect to the interaction scale suppressing the effective interaction vertex. The observables *do* depend on the details of the interaction, and the actual sensitivity to DM parameter space can be lower or higher than what naively estimated.

III *"Simplified" models*
These models include not only the DM state, but also the lightest/most important mediator of DM interaction with the SM (and with itself). They proceed (at least in principle) by constructing the most general (renormalizable, or leading nonrenormalizable terms in power-counting) Lagrangian respecting DM stability constraints + SM exact symmetries (Lorentz and gauge ones), and make sure the accidental and custodial SM symmetries (B,L, flavour, . . .) are not broken too badly. For more details, see e.g. [51]. The crucial parameters are typically four: the masses of DM

and of the mediator, the coupling DM-mediator and the coupling mediator-SM. The parameter space is sufficiently small that often meaningful constraints can be set by combining the requirements on early universe production, collider, direct and indirect (lack of) signatures. This approach is suitable for phenomenological studies, although the links with UV theories is not always clear (a less and less relevant issue, as the hopes to converge any time soon towards some "ultimate" theory seem to fade away). For instance, this attitude was followed when the 750 GeV diphoton bump at LHC [60, 61] was taken seriously in 2016, and the corresponding particle was considered the mediator (see e.g. [62]).

Overall, collider searches are more sensitive to the "mediator" (or partner coloured particles) than to the DM itself. Be also aware that they are not *sufficient* to determine if a particle is the DM, at most can get hints of a particle that might be a good DM candidate. A direct/indirect detection would be needed, anyway, to make sure that it is the same stuff present out there. (just think of how ridiculously short lifetime can be constrained at LHC vs. what needed for the DM!)

B. Direct searches

This approach goes back to pioneering papers like [52].

- **Strategy**: measure recoil energy from elastic scattering of local DM WIMPs with detectors underground (to shield them from cosmic-rays & their induced "activation").
- **Observables**: i) Rate and spectrum of the recoils (possibly different channels!) ii) Time-dependence (modulation) iii) Event Directionality (for future! At R&D stage, requires gaseous detectors. . .)
- **Issues**: separate WIMP-induced recoils from backgrounds (radioactive, cosmic rays,. . .).

For some key formulae, see for instance [53]. For the last issue, note that despite low "noise" reachable in high purity materials *underground* (hence with low cosmic ray induced noise), many phenomena can cause energy deposition, notably traces due to omnipresent radioactive decays (surface around the detector, rocks around, etc.). The largest worry is to separate "e.m.-like" recoils from "nucleon-like" recoils, with the latter much more rare and more similar to the ones expected from WIMPs. Typically, one combines different detector channels (e.g. scintillation vs. phonons/vibrations vs. ionization) and tries to (self)shield the surface events (more likely to be due to background) to improve sensitivity.

The differential rate of events on a target containing N_T target nuclei (of mass number A and mass m_A) is given by

$$R \simeq N_T \, \frac{\rho}{m} \sigma v \,, \tag{68}$$

or in differential form, with respect to recoil energy E_R,

$$\frac{dR}{dE_R} = N_T \frac{\rho}{m} \int_{v_{\min}}^{v_{\max}} \mathrm{d}^3\vec{v} f(v) |\vec{v}| \frac{d\sigma}{dE_R} \tag{69}$$

We thus need to know link between velocities and recoil energy, the dependence of cross-section on the relevant variables, and specify $v_{\min}$ and $v_{\max}$. It is important to make explicit a couple of simplifications which are usually done, linked to the low-energy regime of the phenomena one is looking at:

- In general, only the elastic channel is open for DM-nucleus collisions. There are exceptions, like in the so-called inelastic DM models [54], which we shall not discuss here.
- Like a partial wave analysis readily suggests, we expect that the cross-section is basically isotropic (i.e. in the CM frame, $\mathrm{d}\sigma/\mathrm{d}\cos\theta \sim const.$) with anisotropic terms suppressed by a factor $(v/c)^2 \sim 10^{-6}$.

Let us assume a given DM velocity v in the Lab frame. By elementary mechanics, the recoil energy writes as the kinetic energy of the incoming DM, times a "mass-mismatch" factor (varying between 4 times the mass ratio of the lighter to the heavier, up to 1), times a relative angle-recoil factor, with θ the angle between the outgoing direction and the ingoing one in the CM

$$E_R = \frac{1}{2} m_X v^2 \frac{4\, m_X m_A}{(m_A + m_X)^2} \frac{1-\cos\theta}{2} = \frac{\mu_{XA}^2 v^2}{m_A}(1-\cos\theta) \leq \frac{2\mu_{XA}^2 v^2}{m_A} \equiv E_R^{\max} \,. \tag{70}$$

where the RHS shows that $E_R \leq E_R^{\max} \sim \mathcal{O}(100)$ keV check!, thus requiring low energy detectors! Also, the minimum velocity that can lead to a given E_R is obtained when the angular factor is most favourable, i.e. $1-\cos\theta = 2$, so we can rephrase into $v_{\min}(E_R) = m_A\, E_R/(2\mu_{XA}^2)$.

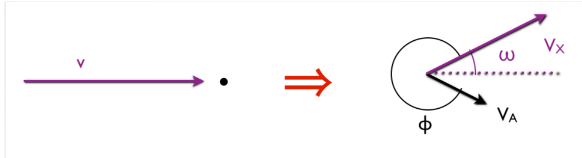

FIG. 2: Auxiliary figure to clarify the kinematics of the exercise (here in the Lab frame).

Exercise: Prove the above. *Hint*: Start from the DM energy in the lab, $m_X v^2/2$, and its momentum in the lab, $p_X = m_X v$. Imposing energy conservation, momentum conservation along the direction of collision, and momentum nulling perpendicular to it, derive

$$E_R = \frac{1}{2} m_X v^2 \frac{4\, m_X m_A}{(m_A + m_X)^2} \cos^2\phi\,. \tag{71}$$

Then, the definition of ϕ implies $\tan\phi = V_A^y/V_A^x$. In the CM frame, the velocity perpendicular to the incoming direction does not change, $W_A^y = V_A^y$, while $W_A^x = V_A^x - V_{CM}$, where by definition of θ, one also has $W_A^y = -V_{CM}\sin\theta$, and $W_A^y = -V_{CM}\cos\theta$, i.e. $\tan\phi = \sin\theta/(1-\cos\theta) = \cot\theta/2 \Rightarrow \cos^2\phi = (1-\cos\theta)/2$.

Then, by using the isotropic nature of the cross-section in the CM frame, and dubbing it $\sigma_0 = \int \mathrm{d}\Omega \mathrm{d}\sigma/\mathrm{d}\cos\theta$,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_R} = \frac{\mathrm{d}\sigma}{\mathrm{d}(-\cos\theta)}\frac{\mathrm{d}(-\cos\theta)}{\mathrm{d}E_R} = \frac{\sigma_0}{2}\frac{2}{E_R^{\max}(v)}\,. \tag{72}$$

The final step left is to integrate over the incoming DM velocity distribution, obtaining ($v_{\min}(E_R) = m_A\, E_R/(2\mu_{XA}^2)$)

$$\frac{dR}{dE_R} = N_T \frac{\rho}{m}\frac{\sigma_0\, m_A}{2\mu_{XA}^2}\mathcal{I}(v_{\min}(E_R))\,, \tag{73}$$

where (the upper cutoff can be usually ignored, and is rather linked to the escape velocity from the Galactic halo)

$$\mathcal{I}(v_{\min}) \equiv \int_{v_{\min}}^{v_{\max}} \mathrm{d}^3\vec{v}\frac{f(v)}{v}\,. \tag{74}$$

For a reference Mawvellian-like velocity distribution, $f(v) \propto \exp(-v^2/(2\sigma^2))$, one has $\mathcal{I}(v_{\min}) \propto \exp(-v_{\min}^2/(2\sigma^2))$, which also leads to the expectation that most recoils are at low-energy,

$$\frac{dR}{dE_R} \propto \exp\left(-\frac{E_R}{E_0}\right)\,, \tag{75}$$

where for simplicity we neglected the (usually sub-leading) dependence upon E_R hidden in σ_0, the elastic scattering cross-section between *DM and the nucleus*. How to express this quantity? First, notice that the elastic scattering between DM and *nucleons* is described in terms of a handful of four-fields operators involving DM and the nucleons ($X\mathcal{O}_1 X N \mathcal{O}_2 N$). There is only a small number of (Gallilean, in the non-relativistic limit) invariant structures and a handful of Hermitian variables (exchanged momentum, spins,. . .) the result can depend upon, allowing one to construct a very efficient effective field theory, see for instance [55]. Actually, once the spin of the DM and the nature of the mediator is fixed, the number of terms that dominate is usually even smaller and "inherited" from the Lorentz-invariant structure of the relativistic DM theory. For instance, for SUSY neutralinos what matters are just: i) the axial vector coupling; ii) the scalar coupling, yielding as leading non-relativistic interactions a spin-dependent and a spin-independent term, respectively. Further, since the momentum of the DM particles is so low that typically their de Broglie wavelength encloses the whole nucleus, DM interacts *coherently* with all nucleons of the nucleus (i.e. compute the modulus square of the relevant matrix element as first summing amplitudes over nucleons and then squaring, not summing the squares of the single-nucleons amplitudes!). This means that in the two cases just mentioned: for the spin-dependent part only nucleon(s) outside completed nuclear shells matter, hence to enhance sensitivity one does not need particularly heavy nuclei, rather nuclei with some "unpaired" nucleons and a relatively large nuclear spin. On the other hand, the spin-indepependent cross section scales (for isospin-blind coupling) as $\propto A^2$ times the single nucleon cross-section, rather than as A times it as one might have naively thought [3].

[3] For large nuclei and sufficiently large E_R, WIMPs start "resolving" the nucleus (their De Broglie wavelength becomes small enough $\lambda \sim (m_A E_R)^{-1/2} \lesssim R_A \simeq 1.2 A^{1/3}$ fm), in which case this simple way to compute the cross-section with the nucleus should be corrected via a multiplicative form-factor $F^2(E_R)$, with F loosely speaking the Fourier transform of the nucleus density. See e.g. [53] for more details.

The expression (73) also allows one to understand the typical $\nu-$shape of the sensitivity curves of direct detectors underground to σ_0 vs. m_X. At high-values of DM mass, $m_X \gg m_A$, $\mu_{XA} \simeq m_A$, the integral does not depend on m_X and only the pre-factor matters, so that the bound is actually on σ_0/m_X (i.e., linear). At low-masses, $m_X \ll m_A$, $\mu_{XA} \simeq m_X$ and $\mathcal{I}(v_{\rm min}) \sim \exp(-E_R m_A/(2\, m_X^2 \sigma^2))$ dominates the mass-dependence, which implies a rapid loss of sensitivity at low-masses, which can only be mitigated by using low-threshold detectors and/or relatively light target nuclei. But remember that for those, one loses sensitivity to the *nucleon* cross-section in coherent scalar scattering, since the A^2 enhancement is small: experimentalists use different targets, so they compare performances by plotting bounds to the nucleon cross-sections! Then, detectors that are sensitive at very low DM masses are not the ones that achieve overall the best sensitivity! Also, the maximum sensitivity of a detector is expected to be (within factors of $\mathcal{O}(1)$) around $m_X \sim m_A$. Note:

- There is a difficulty hidden in the above. What theories often provide are e.g. WIMP-quark couplings and amplitudes. How to link WIMP-parton amplitudes with WIMP-nucleon ones? For that, we need to know the values of the quark currents inside the nucleon, which in turn depend upon non-perturbative QCD. For instance, a more fundamental expression for σ_0 for scalar interactions is

$$\sigma_0 = \frac{4\mu_{XA}^2}{\pi}[\lambda_p Z + \lambda_n (A-Z)]^2 \,, \tag{76}$$

  (note that $\sigma_0 \propto A^2 \sigma_N$, if $f_p = f_n$, as said) where the nucleon couplings are expressed in terms of the quark ones as

$$\frac{\lambda_N}{m_N} = \sum_q f_{q,N} \frac{\lambda_q}{m_q} \,, \tag{77}$$

  and

$$m_N\, f_{q,N} = \langle N | m_q \bar{q} q | N \rangle \,. \tag{78}$$

  Thus, the proportionality coefficients are nothing but the contributions of quark q to the nucleon mass, m_N. These should be deduced from nuclear/hadronic physics and/or lattice QCD, and are still affected by some uncertainties (for some values, see e.g. [56]).

- One needs to know not only the spatial density of DM at the Earth, but also its velocity distribution. DM is often assumed to be distributed as a (truncated) MB in a frame at rest with respect to the Galactic halo. This is nowadays roughly (but only roughly!) shown in halo assembly simulations. Note that this *is not* due to a DM "thermalization". Rather, the heuristic understanding of this phenomenon goes under the name of "violent relaxation" [57], where sudden variations of the potential (like mergers) lead to fast mixing of the phase-space elements of coarse grained f into highest-entropy configuration (at fixed energy), leading to a Maxwell-Boltzmann-like distribution for the *coarse-grained distribution only*. f keeps however some memory, like streams, of the assembly history of the halo: this provide an irreducible "theoretical uncertainty" in f. Also note that the phase-space conservation imposes some constraints on the velocity distribution, *given a spatial DM density*. Under further conditions, like symmetries, this permits a theoretical derivation of $f(v)$, see e.g. [1]. One notable example is the so-called isothermal sphere profile, $\rho \propto r^{-2}$, which leads to a MB distribution under the assumption of isotropy in velocity space.

- A further step omitted above is that the velocity distribution that matters is the one observed *at the Earth* not in the Galactic halo rest frame! To compute what we need, we have to apply a boost to the velocity distribution in the halo accounting for: i) the velocity of the Sun in the Galactic halo. ii) the velocity of the Earth around the Sun. The former effect induces a "wind" of DM particles even in a isotropic halo distribution: for a fixed threshold, the DM event rate is thus expected to be anisotropic, with an enhancement towards the Cygnus constellation (towards which the Sun is moving at about 220 km/s). The latter effect induces a sinusoidal time-modulation of the DM signal, due to Earth revolution, hence of period 1 year, phase peaking around June 2nd and amplitude which depend from the v-distribution (universal!) and the detector (material, threshold...), but usually in the 1-10% range. Both are potential discriminating signatures (with the latter much easier to put in evidence in an experiment). A more extensive discussion can be found in [58].

  Exercise: Check the expressions for the modulated event rate, repeating the derivations in [58].

Current direct DM constraints are most efficient at a few tens of GeV and especially for scalar interactions, where detectors have achieved sensitivies of the order of $\sigma_N \sim (\sigma_0/A^2) \sim \mathcal{O}(10^{-46})\text{cm}^2$! Currently, Xenon-based detectors (more easily scalable, self-shielding, relatively easy to obtain in pure form being a noble element, etc.) lead this race (the latest publication in this domain, which is also the current record-holder in sensitivity, is [59]).

C. Indirect searches

Another way to look for DM is to look for its stable SM byproducts in annihilation (or decay) events. Basically, what was keeping WIMPs in chemical equilibrium in the early universe should still go on today (preferentially in regions with high DM density): given the EW scale mass for WIMPs, we expect energetic SM particles (such as gamma rays, neutrinos, $e^{\pm}$ ...) to be emitted. The spectrum of the particles depend on the model. For a pair of particles i resulting from the annihilation of DM, one has (to a good approximation) the monochromatic spectrum

$$\frac{dN_i}{dE}(E) = 2\delta(E - m_X)\,, \tag{79}$$

since the broadening due to velocity dispersion is small ($v^2/c^2 \lesssim 10^{-5}$). If one is not interested in the particles of the binary final state, but to its fragmentation/decay byproducts (e.g. if i is a muon) it is enough to boost the spectrum of the final particles emitted for i "at rest" by the Lorentz factor $\gamma = m_X/m_i$. For details on this, see the relevant particle physics courses.

But even for a fixed particle physics model of DM (and assuming that one can compute the signal, given the model, which is not always completely true) there are two major challenges in this quest:

i) to know the DM distribution, which is required to know this "exotic" source of energetic particles.

ii) To know the competing astrophysical signals, to separate "DM signal" from "astrophysical background".

To tackle the former, a combination of observational constraints and guidance from simulations are usually employed. To tackle the latter, some general qualitative criteria can be followed, such as focusing charged particle searches in the *antimatter* component of CRs ($e^+, \bar{p}, \overline{D}, \overline{^3\mathrm{He}}$), or to γ-ray lines (for which no astrophysical example is known, above a few tens of MeV). But ultimately, a greatly improved understanding of high-energy astrophysics is needed for further improvements in sensitivity. So, be ready and humble enough to study and learn a lot of astrophysics, if you want to work in this area! This effort is however rewarding: differently from collider or direct detection, in indirect DM studies even negative results in DM searches are often accompanied by *astrophysical* discoveries!

The number of particles of type i per unit energy E per unit volume per unit time injected at position $\mathbf{x}$ at time t via annihilation of a particle X and antiparticle $\bar{X}$,w with average cross section times velocity $\langle\sigma v\rangle$:

$$Q_i(\mathbf{x},t,E) = \frac{dN_i}{dE}(E)\, n_X(\mathbf{x},t) n_{\bar{X}}(\mathbf{x},t)\langle\sigma v\rangle \quad \text{if } X \neq \bar{X}\,, \tag{80}$$

or

$$Q_i(\mathbf{x},t) = \frac{dN_i}{dE}(E)\, \frac{n_X^2(\mathbf{x},t)}{2}\langle\sigma v\rangle \quad \text{if } X = \bar{X}\,, \tag{81}$$

while for a decaying species X with lifetime τ_X

$$Q_i(\mathbf{x},t,E) = \frac{n_X(\mathbf{x},t)}{\tau_X}\frac{dN_i}{dE}(E)\,. \tag{82}$$

Note that

$$\int \mathrm{dE}\frac{dN_i}{dE}(E) = \mathrm{br}_i\,. \tag{83}$$

In case the species X is responsible for the DM abundance (and X and $\bar{X}$ are equally abundant and accounting for half of the DM each, if the particle is not self-conjugated)

$$Q_i(\mathbf{x},t,E) = \frac{\langle\sigma v\rangle}{4\,(2)}\frac{\rho_{\mathrm{DM}}^2(\mathbf{x},t)}{m_{\mathrm{DM}}^2}\frac{dN_i}{dE}(E) \quad \text{annihilation if } X \neq \bar{X}\ (X = \bar{X}) \tag{84}$$

$$Q_i(\mathbf{x},t,E) = \frac{\rho_{\mathrm{DM}}(\mathbf{x},t)}{\tau_{\mathrm{DM}}}\frac{dN_i}{dE}(E) \quad \text{decay}\ . \tag{85}$$

These expressions are usually more practical since one knows (or has constraints on) ρ_{DM}, not on $n_{\mathrm{DM}}^2 = \rho_{\mathrm{DM}}^2/m_{\mathrm{DM}}$. If one seeks the emission term per unit solid angle, the above expressions should be multiplied by a further factor $1/(4\pi)$.

1. From injection to detection - neutral particles

For particles propagating in straight lines, the above source term can lead directly to observables (e.g. "sky maps") via integration along the line of sight (indicated below via the angular variables in Galactic coordinates, b and l). The schematic structure of the differential flux *(number of particles per unit time, energy, surface and solid angle)* is then

$$\frac{\mathrm{d}\Phi_i}{\mathrm{d}E}(E,b,l) = \frac{1}{4\pi}\int_0^\infty Q_i(\mathbf{x},t,E)\, e^{-\tau(E,s,b,l)}\,\mathrm{d}s\ , \tag{86}$$

accounting for possible absorption with optical depth $\tau \sim \sigma\, n\, \ell$. Note that a further factor $1/(4\pi)$ appears, since we seek the flux per unit solid angle, too. For annihilations (assuming self-conjugate candidate):

$$\frac{\mathrm{d}\Phi_i}{\mathrm{d}E}(E,b,l) = \frac{\langle\sigma v\rangle}{8\pi\, m_{\rm DM}^2}\frac{\mathrm{d}N_i}{\mathrm{d}E}(E)\int_0^\infty \rho_{\rm DM}^2(s,b,l)\, e^{-\tau(E,s,b,l)}\,\mathrm{d}s\ , \tag{87}$$

and for decaying DM

$$\frac{\mathrm{d}\Phi_i}{\mathrm{d}E}(E,b,l) = \frac{1}{4\pi\, m_{\rm DM}\,\tau_{\rm DM}}\frac{\mathrm{d}N_i}{\mathrm{d}E}(E)\int_0^\infty \rho_{\rm DM}(s,b,l)\, e^{-\tau(E,s,b,l)}\,\mathrm{d}s\ . \tag{88}$$

For a spherically symmetric DM distribution (as typically assumed for the halo of our Galaxy) one has $\rho_{\rm DM}(s,b,l) = \rho_{\rm DM}[\varrho(s,b,l)]$ where

$$\varrho(s,b,l) = \sqrt{s^2 + R_\odot^2 - 2sR_\odot\cos b\cos l}\,. \tag{89}$$

For astrophysical targets in our "neighborhood" it is customary to normalize to the local DM abundance, near the Sun (and solar distance) so that the signal writes (neglecting absorption)

$$\frac{\mathrm{d}\Phi_i}{\mathrm{d}E}(E,b,l) = \frac{\langle\sigma v\rangle\rho_\odot^2\, R_\odot}{8\pi\, m_{\rm DM}^2}\frac{\mathrm{d}N_i}{\mathrm{d}E}(E) J_{\rm ann}(l,b) \tag{90}$$

where (be aware, sometimes different conventions used!) the J-factor is defined as

$$J_{\rm ann}(l,b) \equiv \int_0^\infty \frac{\rho_{\rm DM}^2(s,b,l)}{\rho_\odot^2}\,\frac{\mathrm{d}s}{R_\odot}\,. \tag{91}$$

Less frequently, one also finds a similar definition for the decay case

$$\frac{\mathrm{d}\Phi_i}{\mathrm{d}E}(E,b,l) = \frac{\rho_\odot\, R_\odot}{4\pi\, m_{\rm DM}\,\tau_{\rm DM}}\frac{\mathrm{d}N_i}{\mathrm{d}E}(E) J_{\rm dec}(l,b) \tag{92}$$

where

$$J_{\rm dec}(l,b) \equiv \int_0^\infty \frac{\rho_{\rm DM}(s,b,l)}{\rho_\odot}\,\frac{\mathrm{d}s}{R_\odot}\,. \tag{93}$$

Note that these "angular-dependence" functions can largely exceed unity: For illustration, a typical expectation for our Galactic halo is reported in Fig. 3.

The above formulae take a special form if applied to cosmological distances, since one has to take into account the redshift effects. In particular, by taking into account that at redshift z, the solid angle element $d\Omega$ and and the radial increment dr determine the proper volume in terms of the comoving one as $dV = (1+z)^3\, r^2 dr d\Omega$ (see [64] for a throughout discussion) and since we are interested in the number of photons collected at the Earth in a time dt_0 and energy interval dE_0 from such a volume, using $dt_0 dE_0 = (1+z)^{-1}dt(1+z)dE$ one has simply

$$\frac{d\Phi_i}{dE} = \frac{\Omega_{\rm DM}\,\rho_{c,0}}{4\pi\, m_{\rm DM}\tau_{\rm DM}}\int_0^\infty dz\,\frac{e^{-\tau(E,z)}}{H(z)}\frac{dN_i}{dE}((1+z)E)\,, \tag{94}$$

for a decaying DM (the factor $(1+z)^3$ from DM evolution exactly cancels the one coming from the proper volume) where $H(z) = \sqrt{\Omega_\Lambda + (1+z)^3(\Omega_{\rm DM}+\Omega_b)}$ in currently favored ΛCDM universe. For the annihilating DM case, one has

$$\frac{d\Phi_i}{dE} = \frac{\langle\sigma v\rangle\Omega_{\rm DM}^2\,\rho_{c,0}^2\, c}{8\pi\, m_{{\rm DM}^2}\tau_{\rm DM}}\int_0^\infty dz\,\frac{(1+z)^3 e^{-\tau(E,z)}}{H(z)}(1+\langle\delta(z)^2\rangle)\frac{dN_i}{dE}((1+z)E)\,, \tag{95}$$

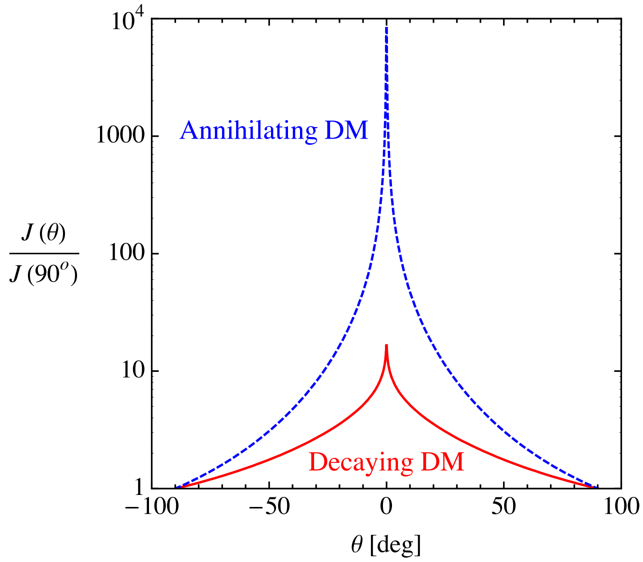

FIG. 3: Plot from [63].

where

$$\langle\delta(z)^2\rangle = \int^{k_{\max}} \frac{\mathrm{d}k}{k}\frac{k^3 P_{\rm NL}(k)}{2\pi^2} \equiv \int^{k_{\max}} \frac{\mathrm{d}k}{k}\Delta_{\rm NL}(k) \tag{96}$$

Notice the dependence from $k_{\max}$, which depends on DM microphysics (e.g. the kinetic decoupling of a WIMP, the initial velocity dispersion of a superWIMP, etc.) besides evolution under gravity. By the way, baryonic effects are likely important, too! For curiosity, you may have a look at [65, 66] to see non-linear power spectra taken from simulations, "reasonable" extrapolations and corresponding estimate for $\langle\delta(z)^2\rangle$, also known as $\zeta(z)$.

For any possible target, we need to know $\rho_{\rm DM}(\mathbf{x},t)$, which enters the above formulae. However, it is also clear that the DM putative signal will be stronger, the bigger the density is. This suggests that we will often need to know $\rho_{\rm DM}$ at places where it departs a lot from the cosmological average value $\Longrightarrow$ Often far from perturbation theory predictions! One exception is the cosmological average signal from DM decay, see Eq. (94). For the annihilating DM case, this dependence is explicit, see Eq. (96).

In general there are many targets we should be interested in, say, for gamma-ray observations. To have an idea of how the DM annihilation sky would look like (without any astrophysical signal competing!) have a look at Fig. 4. Clearly the inner Galaxy looks very promising. But these are simulations: What about observational constraints? For that we need to study things such as the distribution of stars and gas as well as probes of the rotation curve, see e.g. [11] for a recent study. The uncertainties are notable especially in the inner halo due to the fact that baryons are dynamically dominant there (see Fig. 5, taken from [68], to have an idea of the uncertainties). Also bear in mind that the inner Galaxy is densely populated with both diffuse and astrophysical sources of gamma-rays. A compromise between backgrounds and signal, maximizing the "signal-to-noise" ratio, suggests one to look for the signal from some "corona" in the inner halo, see e.g. [69].

Another interesting signal emerging from Fig. 4 is the one from sub-halos (or, in principle, clusters of galaxies). What about signal expected from "a clump", like a satellite of the Milky Way? We will not indulge on that here, but we can estimate its J-factor as

$$J_{\rm dwarf} \propto \frac{1}{\rho_\odot^2 R_\odot}\frac{2}{d^2}\int_0^\infty dr r^2 \rho_{DM}^2(r) \propto \frac{\langle\rho_{\rm DM}^2\rangle_{V_{\rm dw}}}{d^2}, \tag{97}$$

So, remember that for dwarf spheroidal bounds (nominally the best ones currently available on "standard" WIMPs, see [70]) the constraints are affected: by the error on the distance of the object from us, d, and the volume integral of their DM density distribution squared! A good starting point to make yourself an idea of the kind of uncertainties in this business is [70, 71]. Many efforts are going into reducing these uncertainties as well as discovering new satellites, where major advances are expected via forthcoming cosmological surveys. There is at least one public code, CLUMPY [72], which allows to perform the relevant l.o.s. integrations numerically.

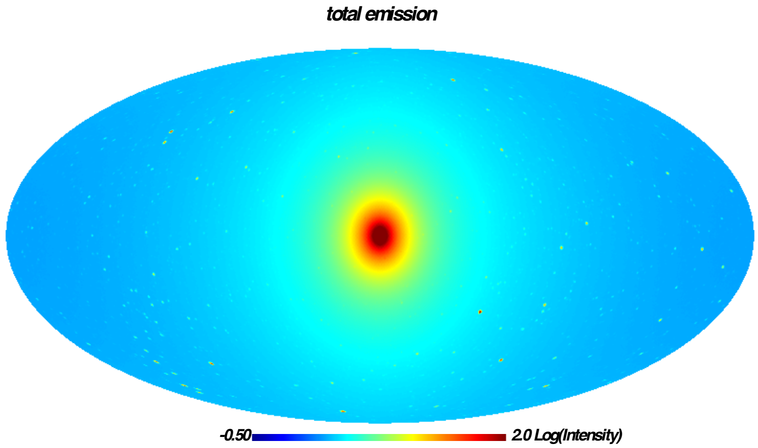

FIG. 4: Example of J-factor (arbitrary normalization) over the whole sky ("Galactic" coordinates): The inner Galactic halo as well as hot-spots corresponding to satellites of the Galaxy are visible from the simulations in [67].

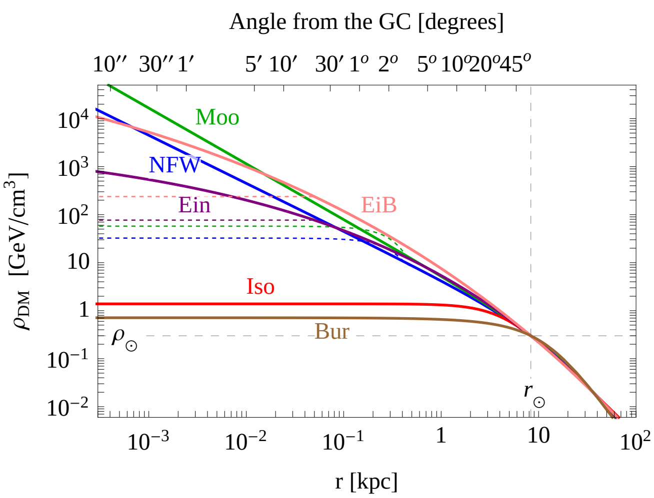

FIG. 5: Illustration of the uncertainties in the DM profile in the inner Galactic halo. From the mini-review in [68].

2. *Neutrinos from the center of the Sun*

The previous considerations also apply to neutrino fluxes, but it's so hard to detect neutrinos that usually the sensitivity to these signals is very poor! Compare for instance the bounds obtained in [73] with the ones coming from [70]. To overcome the detection difficulties, we would like thus to have a boosted signal, by increasing the density of the DM. To do that, we can use a "DM catcher", e.g. the Sun. In fact, DM can collide with solar matter, lose energy, get gravitationally trapped, and eventually sink into its core. There, we can turn neutrino's weak interactions into a strength: for all other SM species, despite annihilation rates are high, no signal gets out, and the collective power of the Sun is useless. Let us sketch here the formalism needer for the solar neutrino DM signal, which is the only channel that does not suffer from this absorption problem, at least up to hundreds of GeV. The following discussion mostly recaps the derivation reported in [74], and following it in detail can be left as as exercise. To get an idea of the annihilation rate $\Gamma_A \sim C_A N^2/2$, the number of

DM particles N captured in the Sun/Earth [4] obeys the following time evolution equation

$$\dot{N} = C - C_A N^2\,, \tag{98}$$

where C is the capture rate and C_A (see below) regulates DM annihilations. *If* both coefficient are constant, solving for $N(t)$ one can derive (Exercise: prove it!)

$$\Gamma_A(t) = \frac{C}{2}\tanh^2\left(\frac{t}{\tau_{\rm eq}}\right)\,,\quad \tau_{\rm eq} = (C\,C_A)^{-1/2}\,. \tag{99}$$

In the limit where steady state is reached within timescales much shorter than the lifetime of the Sun, $t_\odot \simeq 4.6\times 10^9$ yr, one has $\dot{N}=0$, $N=\sqrt{C/C_A}$, and the annihilation rate writes

$$\Gamma_A = \frac{C_A}{2}{N_{\rm eq}}^2 = \frac{C}{2}\,. \tag{100}$$

In this regime, the normalization of the signal only depends on C. More generally, the present value $\Gamma_A(t_\odot)$ depends also on $\tau_{\rm eq}$, i.e. on C_A. C_A can be written in terms of effective volumes $V_{1,2}$, as

$$C_A = \langle\sigma_A v\rangle\frac{V_2}{V_1^2}\,,\quad V_j \simeq \left(\frac{3\,m_{\rm Pl}^2 T_\circ}{2\,j\,m_\chi \rho_\circ}\right)^{3/2} \tag{101}$$

with $T_\circ\,,\rho_\circ$ respectively the central temperature and density of the body under consideration [76], and m_χ the DM mass. Note that the above formula already has an uncertainty due to the only approximate assumption of thermalization, homogeneous conditions of the core, etc., some of which are known to fail in some circumstances, see e.g. [77]. In the other useful limit $\tau_{\rm eq}\gg t_\odot$,

$$\Gamma_A(t_\odot)\approx 0.5\,C^2 C_A\,t_\odot^2\,. \tag{102}$$

This has the following implications for the large $\tau_{\rm eq}$ regime: i) the signal is quadratic in C; ii) there is an additional dependence on C_A (i.e. linear in $\langle\sigma_A v\rangle(\frac{m_\chi\rho_\circ}{T_\circ})^{3/2}$) as well as a quadratic one on $t_\odot$.

Focusing on C, we shall first remind the reader that for most particle physics models (including Kaluza-Klein and most neutralino DM models), the capture in the Sun is actually dominated by *spin-dependent* interactions on hydrogen [78]. This implies among others that we can consider the form factor ≈ 1, which greatly simplifies the formulae. What we should take into account is the probability that a particle, when hitting the Sun at some "depth" (parameterized by the corresponding fraction of the solar mass enclosed, $\mathcal{M}$), drops its initial velocity (drawn by some velocity distribution f_1) to a value below the escape velocity characteristic of that depth.

From rewriting Eqs. (2.8,2.13) of [79] in our notation we get

$$C = \sigma_{0,p}\frac{\rho_\odot\,\epsilon_p\,M_\odot}{m_\chi\,m_p}\int_0^1 {\rm d}\mathcal{M}\,\nu^2(\mathcal{M})\int_0^{u_{\rm max}}{\rm d}u\frac{f_1(u)}{u}\left[1-\frac{u^2}{u_{\rm max}^2}\right]\,, \tag{103}$$

where m_p is the proton mass,

$$u_{\rm max}(\mathcal{M})\equiv\frac{\sqrt{4\,m_\chi\,m_p}}{m_\chi - m_p}\nu(\mathcal{M}) \tag{104}$$

and ν is the escape velocity from the unit shell volume considered, in turn depending on the distance from the center r; to a good approximation [80, 81],

$$\nu^2(r) = v_\circ^2 - \mathcal{M}(r)(v_\circ^2 - v_s^2)\,,$$
$$v_\circ \simeq 1355\,{\rm km/s}\,,\; v_s\simeq 818\,{\rm km/s} \tag{105}$$

The interesting aspect of this channel is that, despite searching for a signal coming from annihilations, it probes scattering cross sections of DM with matter (in particular spin-independent ones), see Eqs. (100,103), and the results are typically compared with direct detection searches, see again [68] for some example. It is also one of the few cases where indirect searches are sensitive also to P-wave annihilating relics. Keep in mind however that a number of astrophysical limitations makes the comparison a bit "sloppy" at the factor 2 level or so, see [74].

Finally, for appropriate calculations, be aware that absorption and neutrino mixing (e.g. via a density matrix approach) should be taken into account, see [82, 83] for works in this direction.

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[4] Typically, the Earth signal is less prominent and we shall neglect that possibility henceforth. See [75] for a computation of this signal.

3. From injection to detection - charged particles

(This is just a very crude sketch! For some general textbook references on cosmic ray astrophysics [84–87].)

For charged particles, one key difficulty is that they do not retain directionality due to deflections in interstellar magnetic fields: CR trajectories are sorts of random-walks which are typically described via a diffusion equation

$$\left\{\frac{\partial\phi}{\partial t}+\nabla\cdot\mathbf{J}_\phi=\mathrm{Source}-\mathrm{Losses}\right\}\oplus\{\mathbf{J}=\mathbf{u}\,\phi-D\nabla\,\phi\}\Longrightarrow\frac{\partial\phi}{\partial t}+\nabla\cdot(\mathbf{u}\,\phi-D\nabla\,\phi)=Q-\frac{\phi}{\tau}\,, \tag{106}$$

where Q is some source term for the species under consideration (including possible sources from DM) and τ accounts for the possible decay of the CR species, but in general at least for its collisional catastrophic losses, even if the species is stable (then $1/\tau\sim\sigma_{\rm inelastic}\,n_{\rm gas}\,v$). Assume that's all there is, and further simplify to 1-D and to constant u, actually even a vanishing one. In the limit where the Galaxy can be considered as a (radially) "infinite" gas thin disk of uniform surface density, sandwiched in a thicker diffusive halo, only the vertical coordinate is relevant—there is a hierarchy $h:H:R\sim 0.1:3:20$ between the thickness of thin disk, size of diffusive halo, and radial size of the disk, justifying this slab model of the Galaxy)—and one has

$$\frac{\partial\phi}{\partial t}-\frac{\partial}{\partial z}\left(D\frac{\partial\phi}{\partial z}\right)+u\frac{\partial\phi}{\partial z}=Q-\frac{\phi}{\tau}\,. \tag{107}$$

Let's look at some solution, to get an idea of what to expect for the "source-to-observable" link.

4. "Leaky box" from slab model

Consider an extended diffusion halo (of radial size much larger than its height $R\gg H$) and consider only diffusion (independent of vertical height) as well as sources and gas (responsible for catastrophic losses) confined to a much thinner"plane" of height h, with density of gas n and injection spectrum per unit time $q_0(p)$. We assume that CR freely escape outside H. By symmetry, the only inhomogeneity may be in z, of course. The steady state transport equation simplifies into

$$-\frac{\partial}{\partial z}\left(D\frac{\partial\phi}{\partial z}\right)=q_0(p)\,h\delta(z)-\sigma\,v\,n\,h\,\phi\delta(z)\,. \tag{108}$$

Following the following solution method in details can be left as an exercise.The previous equation at $z\neq 0$ reduces to $\frac{\partial^2\phi}{\partial z^2}=0$, whose solution is $\phi(z,p)=a(p)+b(p)|z|$. If we denote with ϕ_0 the solution in the plane $\phi_0=\phi(0,p)$, the vanishing at H gives

$$\phi(z,p)=\phi_0(p)(1-|z|/H)\,. \tag{109}$$

An equation for $\phi_0(p)$ can be found by integrating Eq. (108) over a small interval around $z=0$. One finds

$$-2D(p)\left.\frac{\partial\phi}{\partial z}\right|_0+\sigma\,v\,n\,h\,\phi_0=q_0(p)\,h\,. \tag{110}$$

Using Eq. (109) one finds

$$\phi_0(p)=q_0(p)\tau_{\rm eff}(p)\,,\quad\text{where}\quad\tau_{\rm eff}^{-1}(p)=\tau_d^{-1}(p)+\tau_\sigma^{-1}(p)\,. \tag{111}$$

and

$$\tau_d(p)=\frac{H\,h}{2\,D(p)}\approx 10^7\,{\rm yr}\frac{H}{3\,{\rm kpc}}\frac{h}{100\,{\rm pc}}\frac{10^{28}\,{\rm cm^2s^{-1}}}{2\,D}\,,\quad\tau_\sigma(p)=\frac{1}{\sigma\,v\,n}\approx 10^7\,{\rm yr}\left(\frac{1\,{\rm cm}^{-3}}{n}\right)\left(\frac{100\,{\rm mb}}{\sigma}\right). \tag{112}$$

The above solution *in the plane* is equivalent to the so-called "leaky box" model solution.

5. Secondary over primary

Let us apply the previous equation to the case of secondaries, i.e. nuclei only produced by spallation during propagation. This is believed to be the case for instance of Boron, Lithium, and Beryllium, which are very rare in typical thermal astrophysical environments like the Solar wind, but sizable in cosmic ray abundances. The distribution of secondaries in the plane, $\phi_S(p)$ is sourced by the injected nuclides per unit time, i.e. $q_0(p) \to \phi_P/\tau_{\sigma_{P\to S}}$, with ϕ_P being the primary population. Hence we obtain the solution for the ratio of primary to secondary distribution, assuming that the effective propagation time is species-independent

$$\frac{\phi_S(p)}{\phi_P(p)} \simeq \frac{\tau_{\mathrm{eff},P}}{\tau_{\sigma_{P\to S}}} \simeq \frac{\sigma_{P\to S}\, v\, n\, H\, h}{2\, D(p)}. \tag{113}$$

where the second relation holds if collisions are subdominant with respect to diffusion. In principle, n and h can be inferred by independent means (e.g. 21 cm maps, CO lines, surveys for stellar disc thickness...) and from this ratio (for instance, Boron-to-Carbon ratio in CRs) one can thus gauge the value of the "diffusive" ratio H/D, as well as of its energy dependence.

6. Proxy for a primary production

For a DM origin, the production (e.g. of antiprotons) is of a *primary* origin, and way more uniform, notably vertically, than the simple model for astrophysical secondaries considered above. A more appropriate (although geometrically not quantitatively correct!) toy model replacing Eq. (111) is

$$-\frac{\partial}{\partial z}\left(D\frac{\partial \phi}{\partial z}\right) = q_{\mathrm{DM}}(p) - \sigma\, v\, n\, h\, \phi\, \delta(z)\,. \tag{114}$$

I invite you to show as a simple exercise that it follows

$$\frac{1}{\tau_{\mathrm{eff}}(p)}\phi_0 = \frac{H}{h}\, q(p)\,, \tag{115}$$

where $\tau_{\mathrm{eff}}(p)$ is the same defined in Eq. (111). Note that now the expected antiproton signal from DM, say, depends on one extra parameter, that we can take as H (This scaling is physical, because it's linear in the size of the volume from which one collects injected particles.) This means that antimatter signal at the Earth is much more uncertain!

This simple example suggests the following: imagine that the actual flux at the Earth is made of two contributions: astrophysical one, plus a DM one. Even if the respective source terms were exactly known (not true, think for instance of cross-section uncertainties), you have to account for propagation effects. And even if the "astrophysical" ones were under control (e.g. B/C ratio, etc.), the possibility to determine e.g. the ratio D/H from secondary to primary ratio observations as shown above (and there are errors associated to that!), the DM contribution is subject to further *astrophysical/propagation* uncertainties, which are more difficult to reduce, since they depend differently on the astrophysical parameters. Although one has some handles (such as radioactive CRs for the above example of H determination), this "propagation problem" is one of the main difficulties to keep in mind associated to these otherwise promising searches (see [88] for a recent example).

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