# Petnica School of Cosmology

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# Particle Physics and the Early Universe

Lecture Notes

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# PARTICLE PHYSICS and THE EARLY UNIVERSE

Why, in cosmology, should we be interested in the details of the particle physics dynamics which happen at a very microscopical scale?

in radiation domination $\longrightarrow$ $H = \frac{1}{2t} \simeq 1{,}66 \frac{\sqrt{g_*}}{M_{Pl}} T^2 \quad \Longrightarrow \quad t \sim \left(\frac{T}{\text{MeV}}\right)^{-2}$ sec

At early times in the age of the Universe correspond high temperatures, that is high mean energies of the microscopic degrees of freedom in the thermal bath.
To understand the evolution of the Universe, therefore, we need to know which are the relevant degrees of freedom at each energy scale and which are the processes that allow the different species to remain (or not) in thermal equilibrium

Particle physics is the study of the dynamics of such microscopical dof at the different energy scales

## Program of the Lectures

- Introduction to the Standard Model
- Neutrino decoupling
- Dark matter freeze out
- Baryogenesis

DISCLAIMER. In these lectures $\hbar = c = k_B = \mu_0 = 1$, and often $2 = 1$ !!

# Warming-Up

$eV \longleftrightarrow T \sim k_B E$ in K ?

$eV \longleftrightarrow$ Joule?

$1\,eV \cong 1{,}602 \times 10^{-19}$ J

$1\,eV \simeq 11\,604$ K $\quad (\sim 10^4$ K$)$

The center of mass energy of the LHC has been 8 TeV. In a few years it will become 14 TeV

$1$ TeV $= 10^3$ GeV

- Exercise 1: compute in Joule the energy of a 7 TeV proton and compare it with the kinetic energy of an everyday object.
  Do the same for $10^5$ protons (the number of protons in each bunch at LHC)

## Some important scales:

$T_{now} = 2{,}735\,K \sim 0{,}236 \times 10^{-3}\,eV$

$m_\nu \sim eV \sim 10^4\,K$

$E_{atom} \sim 10\,eV \sim 10^5\,K$

$m_e \simeq 0{,}511$ MeV $\qquad (m_\mu \simeq 106$ MeV$)$

what is the mass of a proton in grams?

$m_\pi \simeq 135$ MeV $\leftarrow$ Lightest hadrons

$m_n - m_p \simeq 939{,}5 - 938{,}2 \simeq 1{,}3$ MeV $\longrightarrow t \sim 1$ sec

$m_p \sim 1$ GeV

Up to here it's physics that WE KNOW, often with a very good degree of accuracy

$E_{WEAK} \sim 10^2$ GeV

$f_a \sim 10^{10}$ GeV

These are more speculative ideas, even though well motivated

$E_{GUT} \sim 10^{16}$ GeV

$M_{Pl} \sim 10^{19}$ GeV

# STANDARD MODEL

The dynamics of the microscopical degrees of freedom at the energies tested up to date is described in the framework of Quantum Field Theory
It is an intrinsically quantistic and relativistic description

$$\left.\begin{array}{l} S \sim \hbar \;,\; v \ll c \;\Rightarrow\; QM \\ S \gg \hbar \;,\; v \lesssim c \;\Rightarrow\; SR \end{array}\right\} \; S \sim \hbar \;\&\; v \lesssim c \;\Rightarrow\; QFT$$

To each degree of freedom is associated a field $\varphi(x^\mu)$, the excitations of such field are the particles

The fields are characterized by their representation under the Lorentz group (spin):

- SCALARS - spin 0
  A real scalar field has 1 d.o.f. A complex one has two

- FERMIONS - spin ½
  Fermions are characterized by the chirality $\overset{v}{\uparrow}\underset{\text{spin}}{\Uparrow}$ $\overset{v}{\uparrow}\underset{\text{spin}}{\Downarrow}$ $(\psi_L, \bar{\psi}_L) \oplus (\psi_R, \bar{\psi}_R)$
  Each chirality has 2 dof (the field + its antiparticle)
  A massive fermion (non-Majorana) needs to have both chiralities 4 dof

- VECTORS - spin 1
  The dof of a vector field $V_\mu(x)$ are described by the polarization vector $\varepsilon_\mu$
  - 2 dof if massless $\rightarrow$ needs gauge invariance
  - 3 dof if massive

## Symmetries

A system has a symmetry if there exist a transformation which leaves it unchanged.
A QFT has a symm if $\exists$ a transformation of the fields which leaves the functional integral (classically the action) invariant
To each continuus symmetry it's associated a conserved current (Noether theorem)

GLOBAL SYMMETRIES the parameter of the transformation is a constant

### U(1) SYMMETRY

electron. $\psi \rightarrow e^{i e \alpha}\psi$, $\bar{\psi} \rightarrow e^{-i e \alpha}\bar{\psi}$ $\qquad \mathcal{L} = i\bar{\psi}\partial_\mu\gamma^\mu\psi - m\bar{\psi}\psi \rightarrow$ is invariant Conservation of charge and lepton number

LOCAL SYMMETRIES $\rightarrow$ GAUGE SYMMETRIES

The parameter is a LOCAL function of the spacetime position $\alpha \rightarrow \alpha(x^\mu)$.
$\mathcal{L}$ is no-more invariant. To make it invariant we need to add a massless spin-1 field the "photon" $A_\mu$, and promote the derivative to a COVARIANT one $\partial_\mu \rightarrow D_\mu \equiv \partial_\mu - ieA_\mu$

$$\mathcal{L}^{QED} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Symmetries can also be "non abelian", for example $SU(2)$:

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}. \quad \text{Take } U \in SU(2), \quad U(x) = \exp\left(i g \alpha^a(x) \sigma^a\right)$$

Pauli Matrices

$$Q \to U(x) Q \,, \quad \bar{Q} \to U^\dagger(x) Q \quad \text{and} \quad U^\dagger(x) U(x) = \mathbb{1}$$

In this case one needs to add 3 gauge bosons (one for each generator) $W^a_\mu(x)$

$$\boxed{\begin{array}{c} \text{SM Gauge Symmetries} \\ SU(3)_c \otimes \underbrace{SU(2)_L \otimes U(1)_Y}_{\text{Electroweak Interactions}} \end{array}}$$

Color
"strong interactions"

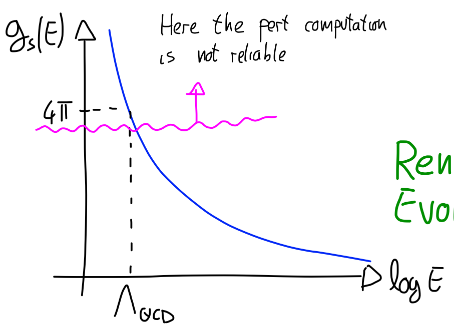
## $SU(3)_c$ - QUANTUM CHROMODYNAMICS QCD

$SU(3)$ has 8 generators $\Rightarrow$ 8 gauge bosons $G^A_\mu$ gluons

Quarks are in the fundamental representation of $SU(3)$ $\mathbf{3}$

$$\Psi = \begin{pmatrix} \psi_R \\ \psi_G \\ \psi_B \end{pmatrix}$$

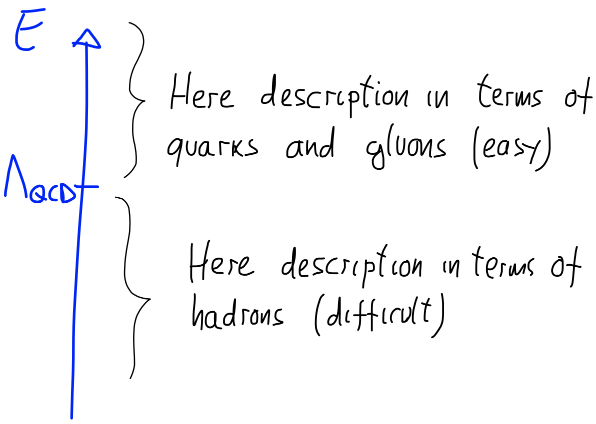
Gauge Coupling $g_s$
Every parameter in a QFT evolves with the energy, it depends on the characteristic energy in the process considered



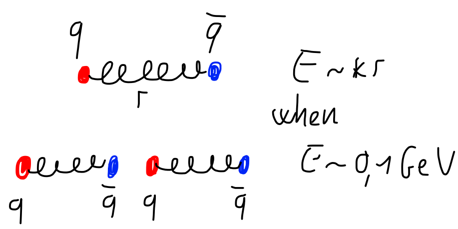

Renormalization Group Evolution

- At high energy the theory is weakly coupled. $g_s \ll 4\pi$
- At energies $E \lesssim \Lambda_{QCD} \sim \text{GeV}$ $g_s$ becomes very strong, the theory becomes non-perturbative

$\boxed{\text{The only asymptotic states are COLOR SINGLET bound states of quarks, antiquarks and gluons HADRONS}}$ $\Leftarrow$ CONFINEMENT




CONFINEMENT



# $SU(2)_L \times U(1)_Y$ Electroweak Symmetry

3 generators: $\sigma^a$ — $W^a_\mu$, $a = 1,2,3$

1 generator — $B_\mu$ HYPERCHARGE

These are weakly coupled interactions at all energies

The vacuum is not invariant under these transformations the symmetry is spontaneously broken, the only remaining unbroken symmetry is the electromagnetic $U(1)_{em}$

electric charge $Q = \frac{\sigma^3}{2} + Y$

The Higgs takes a non-zero vacuum expectation value $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}_{Y=\frac{1}{2}} \Rightarrow \langle 0|H|0\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$,

$v = 246$ GeV

Electroweak symmetry breaking (EWSB)

$$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{em}$$

$\{W^3, B\} \to \{Z, A\}$ — Mass eigenstates

$$\left\{ \begin{array}{l} m_W \simeq 80 \text{ GeV} \\ m_Z \simeq 91 \text{ GeV} \\ m_A = 0 \end{array} \right.$$

# Elementary Particles

## Spin ½

| | $SU(3)_c$ | $SU(2)_L$ | $U(1)_Y$ | $\to Q = T^3_L + Y$ |
|---|---|---|---|---|
| quarks | | | | |
| $\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix}$ | 3 | 2 | 1/6 | 2/3, -1/3 |
| $u_R$ $c_R$ $t_R$ | 3 | 1 | 2/3 | 2/3 |
| $d_R$ $s_R$ $b_R$ | 3 | 1 | -1/3 | -1/3 |
| leptons | | | | |
| $\begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix}$ | 1 | 2 | -1/2 | 0, -1 |
| $e_R$ $\mu_R$ $\tau_R$ | 1 | 1 | -1 | -1 |

} All the hadrons are made of these particles: proton, neutron, pions, kaons, etc.

This classification of quarks and leptons is repeated in 3 generations of heavier "copies"

## Spin 0

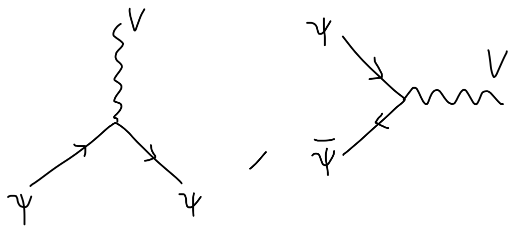
| | $SU(3)_c$ | $SU(2)_L$ | $U(1)_Y$ | |
|---|---|---|---|---|
| Higgs $H$ | 1 | 2 | 1/2 | $h \to Q = 0$ |

Only elementary scalar

# INTERACTIONS

## Gauge Interactions

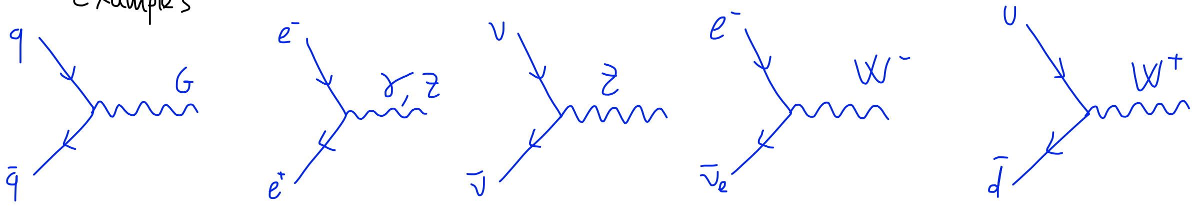
### FERMION - FERMION - GAUGE BOSON



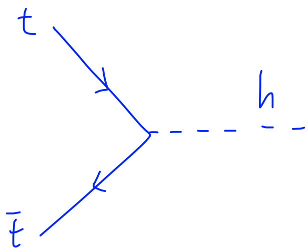

$\sim g_i$

Couples a gauge boson of a gauge symmetry with two fermions in the same representation (fundamental) of the same symmetry group

Examples

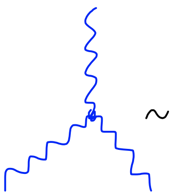



### 2 Fermions and Higgs (Yukawa interactions)

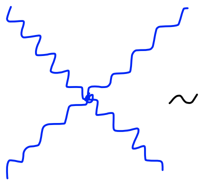



$\sim \frac{m_t}{v}$

### 3 and 4 $SU(2)_L$ and $SU(3)_c$ gauge bosons



$\sim g$

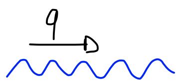


$\sim g^2$

### Other interactions with Higgs and gauge bosons (or only Higgs)

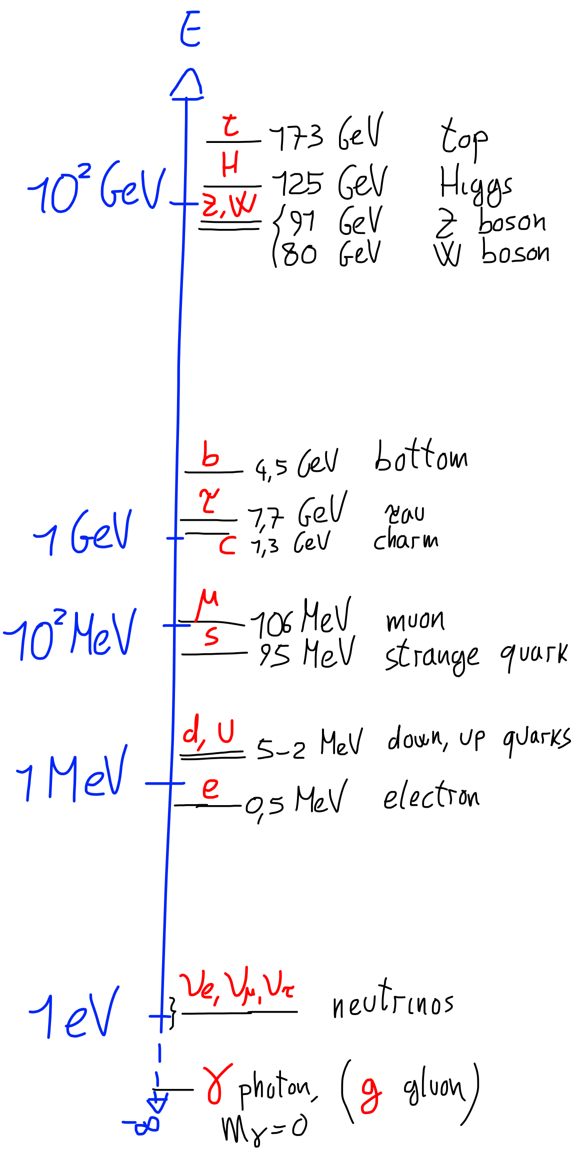
## PROPAGATORS

Each internal line counts as

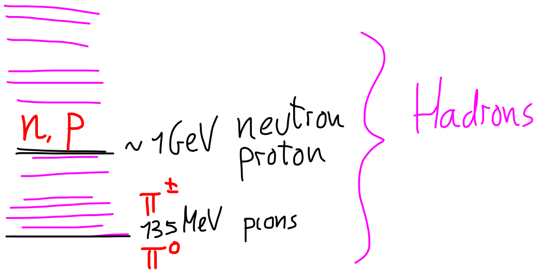



$$\propto \frac{1}{q^2 - m_X^2}$$

# Particle Spectrum of the SM




**Note** For energies $E \gg 1$ GeV, the fundamental degrees of freedom are not the hadrons but quarks and gluons




> **Exercise**
> Compute the relativistic dof of the SM at an energy
> - $E = 1$ TeV
> - $E = 5$ MeV
> - $E = 0{,}1$ MeV

The only parameters of the theory are:

- gauge couplings $g_s,\ g,\ g'$
- Higgs vev. and auto-interaction $v,\ \lambda \leftrightarrow m_h$
- Quark and lepton masses and mixings

> **Note**
> Neutrino masses are a success of the SM as an EFT
> $$\mathcal{L} \supset \frac{1}{\Lambda}(\bar{l}_L^c H)(l_L H)$$

## Open problems of the SM

- Why $v \ll M_{Pl}$ ? $\Longrightarrow$ **Hierarchy Problem**
- Origin of flavor

$\Downarrow$

Expectation to see new physics at the LHC

- Dark Matter
- Cosmological constant
- Baryogenesis
- Inflation

# Departures from Thermal Equilibrium

The Universe has all the structures we observe thanks to departures from thermal equilibrium. Otherwise its state could be specified only by its temperature $T$.
We define $T$ as the temperature of photons in the thermal bath $T \equiv T_\gamma$.
Therefore a specie must be coupled to photons to maintain thermal eq: $(+) \leftrightarrows \gamma\gamma$, or to another specie which in turn is strongly coupled to photons, eg $\nu\bar{\nu} \rightleftarrows e^+e^- \rightleftarrows \gamma\gamma$

The key to understand how and when a specie decouples from the thermal bath is to compare the interaction rate $\Gamma$ with the expansion rate $H = \frac{\dot{a}}{a}$

If the interactions among particles are "fast enough" (with respect to the typical timescale of expansion $\tau \sim H^{-1}$) then the species are coupled strongly with each other and follow together the decreasing temperature. This happens if $\Gamma > H$

How this happens in detail is described by the Boltzmann equation. For the moment we can convince ourselves that it must be so with a simple computation.

Let's assume that $\Gamma(T) = \Gamma_0 T^n$ ← constant $\left(\Gamma \sim \Gamma(E) \to \Gamma(T)\right)$ ← this is often the case

$$T \propto a^{-1} \Rightarrow H = \frac{\dot{a}}{a} = -\frac{\dot{T}}{T} = 1{,}66\, g_*^{1/2} \frac{T^2}{M_{Pl}} = c\, T^2 \Rightarrow \frac{dT}{dt} = -c T^3$$

The number of interactions from a time $t$ to $t \to \infty$ is given by

$$N_{int} = \int_t^\infty \Gamma(t')\, dt' = \int_{T(t)}^0 \Gamma(T') \left(\frac{dT'}{dt}\right)^{-1} dT' = \frac{\Gamma_0}{c} \int_0^T T'^{\,n-3}\, dT' = \left.\frac{\Gamma(t)}{H(t)}\right|_t \frac{1}{n-2}$$

We see that for $n > 2$ (as it is in all cases of interest), from the time $t$ when $\Gamma(t) \leq H(t)$, $N_{int} < 1 \Rightarrow$ the particle doesn't interact for the rest of the history of the Universe

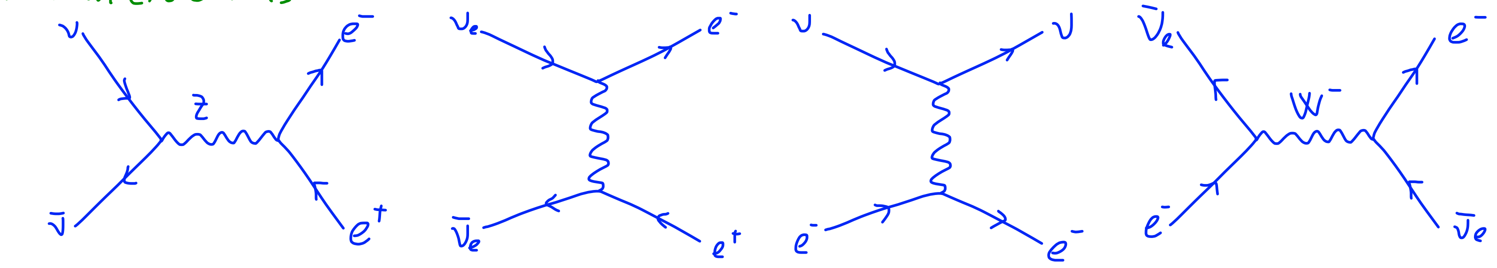
# Neutrino Decoupling

At temperatures $T \lesssim 50$ MeV the relativistic species still present in the plasma are
$e^+, e^-, \nu_{e,\mu,\tau}, \gamma \implies g_* = 2 + \frac{7}{8}(3\times 2 + 4) = \frac{43}{4}$

The interactions that can keep neutrinos in thermal equilibrium with the thermal bath are

$$\nu\bar{\nu} \leftrightarrow e^+e^- \, , \qquad \nu e \leftrightarrow \nu e$$

SM interactions

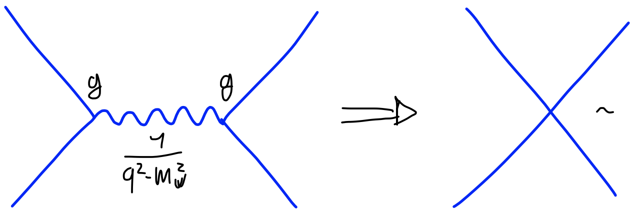



The intermediate "wiggly" line represents the propagator of the gauge boson.
Its contribution to the amplitude is:

$$A \propto g^2 \frac{1}{q^2 - m_V^2} \implies$$

coupling at the vertex ($g^2$), momentum ($q^2$)

If the typical energy of the scattering is $E \ll M_W, M_Z$ then we can expand the propagator for small $q^2$ keeping only the leading term




$$\sim \frac{g^2}{M_W^2} \sim G_F$$

$G_F \simeq 1.16 \times 10^{-5}\ \mathrm{GeV}^{-2}$

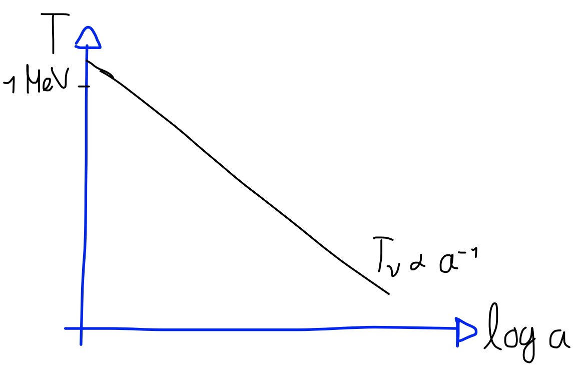
The cross section of the process will be $\sigma \propto |A|^2 \propto G_F^2 T^2$

By dimensional analysis $[\sigma] = E^{-2}$, $[G_F^2] = E^{-4}$ and only relevant energy scale is the temp $T$

$$\left.\begin{aligned} \Gamma &= \langle \sigma v \rangle n \sim G_F^2 T^5 \\ H &\sim g_*^{1/2} \frac{T^2}{M_{Pl}} \end{aligned}\right\} \quad \Gamma \sim H \implies T \simeq \left( \frac{g_*^{1/2}}{G_F^2 M_{Pl}} \right)^{1/3} \simeq g_*^{1/2}\ \mathrm{MeV}$$

Below $T \simeq 1$ MeV neutrinos decouple from the plasma and their temperature will decrease as

$$T_\nu \propto a^{-1} \quad \text{as a free streaming relativistic specie.}$$



At $T \sim m_e \cong 0{,}5$ MeV, electrons become non relativistic $\begin{cases} e^+e^- \to \gamma\gamma & \text{still happens} \\ \gamma\gamma \to e^+e^- & \text{is now forbidden (low } E) \end{cases}$

The electron number density decreases exponentially, the electrons give their entropy to photons

$$n_e^{NR} = g\left(\frac{mT}{2\pi}\right)^{3/2} \exp\left(-\frac{m}{T}\right)$$

To see what happens to photons we use Entropy conservation $\frac{d}{dT}(a^3 s) \cong 0$ (if $\mu \ll T$)

Before electrons become non-relativistic, the entropy of the species in th eq ($e^\pm$ & $\gamma$) is

$$s = \frac{2\pi^2}{45} g_{*s}(T)\, T^3, \qquad \frac{(aT_\gamma)^3_{after}}{(aT)^3_{before}} = \frac{g^s_{*\,bef}}{g^s_{*\,aft}} = \frac{2+\frac{7}{8}4}{2} = \frac{11}{4}$$

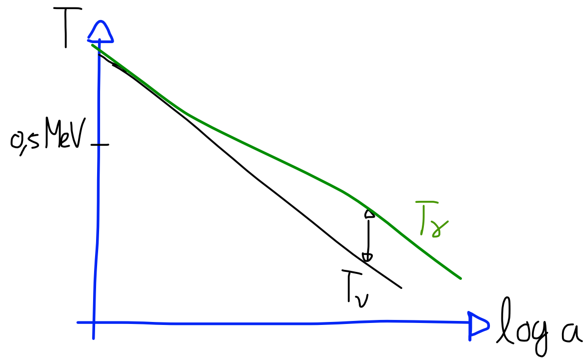
In this period when $e^\pm$ become non relativistic $T_\gamma$ dereases more slowly than $a^{-1}$
When this time ends, the behavior will go back to $T_\gamma \propto a^{-1}$

Neutrinos instead are already decoupled, so they don't receive this entropy injection For them

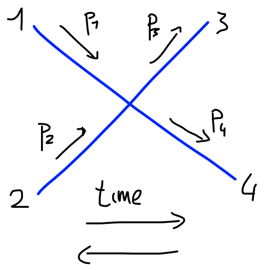
$$(a\,T_\nu)_{after} = (a\,T_\nu)_{before} = (a\,T_\gamma)_{before}$$

$$\Rightarrow (a\,T_\nu)_{now} = \left(\frac{11}{4}\right)^{-1/3} (a\,T_\gamma)_{now}$$

$$T_\nu^{now} = \left(\frac{11}{4}\right)^{-1/3} T_\gamma^{now} \simeq 1{,}96 \text{ K}$$



# Boltzmann Equation




Let's suppose we want to study the abundance of a specie "1" $n_1$
Also, the only process which affects its abundance is $1+2 \rightleftarrows 3+4$

$$n_1 = g\int \frac{d^3p}{(2\pi)^3} f_1(E,t) \leftarrow$$ in homogeneous and isotropic spacetime

$$= \frac{4\pi g}{(2\pi)^3}\int dp\, p^2 f_1(E,t)$$

$$\underbrace{\hat{L}}_{\substack{\text{dynamics} \\ \text{Liouville operator}}}[f_1] = \underbrace{\hat{C}}_{\substack{\text{Interactions} \\ \text{Collision operator}}}[f_{1,2,}\ ]$$

Classically $\quad f = f(\vec{v},\vec{x},t)$

$$\hat{L}_{NR} = \frac{d}{dt} + \frac{d\vec{x}}{dt}\cdot\vec{\nabla}_x + \frac{d\vec{v}}{dt}\cdot\vec{\nabla}_v = \frac{d}{dt} + \vec{v}\,\vec{\nabla}_x + \frac{\vec{F}}{m}\cdot\vec{\nabla}_v$$

In GR $\quad \hat{L} = K^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} K^\beta K^\gamma \frac{\partial}{\partial K^\alpha} \xRightarrow{\text{FRW}} E\frac{\partial}{\partial t} - \frac{\dot{a}}{a}|\vec{K}|^2\frac{\partial}{\partial E}$

The LHS of Boltzmann's eq is, after $\times\, g\int\frac{d^3p}{(2\pi)^3}\frac{1}{E}$,

$$\frac{dn_1}{dt} - H\frac{g}{(2\pi)^3}\int d^3K \frac{|\vec{K}|^2}{E}\frac{\partial f_1}{\partial E} = \dot{n}_1 - H\frac{4\pi g}{(2\pi)^3}\int_0^\infty dK \frac{K^4}{E}\underbrace{\frac{\partial K}{\partial E}}_{=\frac{E}{K}}\frac{\partial f_1}{\partial K} = \dot{n}_1 - H\frac{4\pi g}{(2\pi)^3}\int dK\, K^3\frac{\partial f_1}{\partial K} =$$

$$= \dot{n}_1 + 3Hn_1 = a^{-3}\frac{d}{dt}(n_1 a^3)$$

The RHS is, for $1+2 \rightleftarrows 3+4$ :
$\begin{matrix} + & B \\ - & F \end{matrix}$

$$g\int\frac{d^3K}{(2\pi)^3}\frac{1}{E}\hat{C}[f] = -\int\prod_{i=1}^4 \frac{g_i}{(2\pi)^3}\frac{d^3K_i}{2E_i}\left\{|A|^2_{I\to F}\, f_1 f_2 (1\pm f_3)(1\pm f_4) - |A|^2_{F\to I}\, f_3 f_4 (1\pm f_1)(1\pm f_2)\right\}\times$$

$$\times (2\pi)^4\delta^4(K_I - K_F)$$

- Are a set of coupled integral-partial differential equations

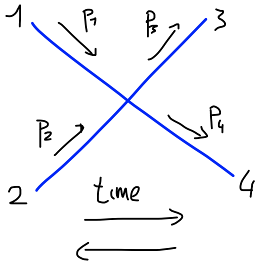
## Simplifications

- for each process we need to consider only a few species
- CP conservation (T conservation) $\quad |A|^2_{\rightarrow} = |A|^2_{\leftarrow} = |A|^2$
- Neglect Bose enhancement / Fermi blocking
- Kinetic equilibrium Scatterings like $1+3 \rightleftarrows 1+3$ etc are always rapid ($n_3$ is big), this enforces $f_i$ to take the form of Bose/Fermi distributions

$$f_i = \left(e^{\frac{E_i-\mu_i}{T}} \pm 1\right)^{-1} \Rightarrow$$ The only variables which evolve are $T$ and $\mu$

If also annihilations were in th eq then $\mu$ = chemical potential, and $\mu_1+\mu_2 = \mu_3+\mu_4$

- Bose / Fermi distributions $\rightarrow$ Maxwell / Boltzmann $\quad f_i(E,t) \rightarrow e^{-\frac{E-\mu}{T}}$




The Boltzmann eq for an expanding Universe is

$$a^{-3}\frac{d}{dt}(n_1 a^3) = \underbrace{\prod_{i=1}^{4}\int\frac{d^3p_i}{(2\pi)^3\, 2E_i}}_{\text{Integral in phase space}}\ \underbrace{(2\pi)^4\,\delta^4(p_1^\mu+p_2^\mu-p_3^\mu-p_4^\mu)}_{\text{Conservation of momentum}}\ |A|^2\ \{f_3 f_4 - f_1 f_2\}$$

neglecting Bose enhancement and Pauli blocking

$N_1$ particles in the comoving volume $a^3$

scattering amplitude Determined by fundamental physics

$$n_i = g_i\int\frac{d^3p}{(2\pi)^3}\, f_i(E)$$

- If there are no collisions $(|A|=0) \Rightarrow \dfrac{d(n_1 a^3)}{dt} = 0 \Rightarrow \boxed{n_1 \propto a^{-3}}$

- For simplicity we approximate $\quad f_i \simeq e^{-\frac{E_i}{T}}\, e^{\frac{\mu_i}{T}}$

$$\{f_3 f_4 - f_1 f_2\} \cong e^{-(E_1+E_2)/T}\left\{e^{(\mu_3+\mu_4)/T} - e^{(\mu_1+\mu_2)/T}\right\}$$

- We relate $\mu_i \rightarrow n_i = g_i\, e^{\mu_i/T}\displaystyle\int\frac{d^3p}{(2\pi)^3}\, e^{-E_i/T}$.

Define the equilibrium $\quad n_i^{(0)} \equiv g_i\displaystyle\int\frac{d^3p}{(2\pi)^3}\, e^{-E_i/T}$

$$\left.\right\}\quad e^{\mu_i/T} = \frac{n_i}{n_i^{(0)}}$$

- Finally, define

$$\langle\sigma v\rangle \equiv \frac{1}{n_1^{(0)} n_2^{(0)}}\prod_{i=1}^{4}\int\frac{d^3p_i}{(2\pi)^3\, 2E_i}\ (2\pi)^4\,\delta^4(p_1^\mu+p_2^\mu-p_3^\mu-p_4^\mu)\ |A|^2\ e^{-(E_1+E_2)/T}$$

We can now write the Boltzmann equation as

$$\boxed{a^{-3}\frac{d}{dt}(n_1 a^3) = n_1^{(0)} n_2^{(0)}\,\langle\sigma v\rangle\left\{\frac{n_3\, n_4}{n_3^{(0)}\, n_4^{(0)}} - \frac{n_1\, n_2}{n_1^{(0)}\, n_2^{(0)}}\right\}}$$

$$\boxed{a^{-3}\frac{d}{dt}(n_1 a^3) = n_1^{(0)} n_2^{(0)} \langle\sigma v\rangle \left\{\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}}\right\}}$$

The LHS is of order $\sim \frac{n_1}{t} \sim n_1 H$

The RHS is of order $\sim n_1 \Gamma$, where $\Gamma \equiv n_2^{(0)} \langle\sigma v\rangle$ Interaction rate

- If $\Gamma \gg H$, for the equality to hold, the RHS must cancel

Chemical Equilibrium $\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} \simeq \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \Rightarrow$ if 3,4 are in equilibrium $\Rightarrow n_1 n_2 \simeq n_1^{(0)} n_2^{(0)}$

$$\boxed{\mu_3 + \mu_4 = \mu_1 + \mu_2}$$

Let's now specialize to the case $\chi + \bar{\chi} \rightleftarrows p + \bar{p}$ where $p, \bar{p}$ are in equilibrium

$$a^{-3}\frac{d}{dt}(n_\chi a^3) = -\langle\sigma v\rangle \left\{ n_\chi^2 - (n_\chi^{eq})^2 \right\} \quad \leftarrow \text{assuming } n_\chi = n_{\bar{\chi}}$$

Define $y_\chi \equiv \frac{n_\chi}{s}$ $\Rightarrow a^{-3}\frac{d}{dt}(y_\chi s a^3) = s\dot{y}_\chi + a^{-3} y_\chi \underbrace{\frac{d}{dt}(s a^3)}_{=0} = s\dot{y}_\chi$

$$\dot{y}_\chi = -s\langle\sigma v\rangle \left( y_\chi^2 - (y_\chi^{eq})^2 \right)$$

Now we change variable $t \to x \equiv \frac{m_\chi}{T}$ $\qquad H = \frac{\dot{a}}{a} = -\frac{\dot{T}}{T} \qquad (T \propto a^{-1})$

$$\frac{dy_\chi}{dt} = \frac{dy_\chi}{dx}\frac{dx}{dT}\frac{dT}{dt} = \frac{dy_\chi}{dx}\left(-\frac{x}{T}\right)(-TH) = xH\frac{dy_\chi}{dx}$$

$$\Longrightarrow \frac{x}{y_\chi^{eq}}\frac{dy_\chi}{dx} = -\frac{s\, y_\chi^{eq}\langle\sigma v\rangle}{H}\left[\left(\frac{y_\chi}{y_\chi^{eq}}\right)^2 - 1\right] = \frac{\Gamma_\chi}{H}\left[\left(\frac{y_\chi}{y_\chi^{eq}}\right)^2 - 1\right]$$

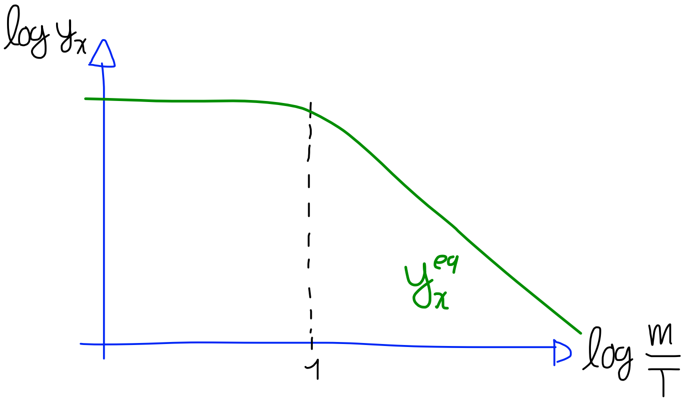
If $\Gamma_\chi \gg H$ at high $T \gg m_\chi \Rightarrow y_\chi \simeq y_\chi^{eq}$. As $T$ drops, $n_\chi$ decreases as $n_\chi^{eq} \propto T^3$

If $T \ll m_\chi \to n_\chi^{eq} \propto (T m_\chi)^{3/2} \exp(-m_\chi/T) \Rightarrow$ exponentially suppressed.

So, at some temperature $T_F$ $(x_F)$ the specie will reach a regime $\frac{\Gamma}{H} \lesssim 1$

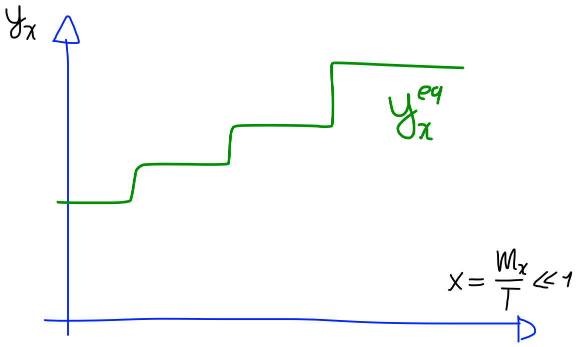
Define the FREEZE OUT temperature $T_F$ (or $x_F$) as $\Gamma(x_F) = H(x_F)$

- For $x \ll x_F \Rightarrow y_\chi(x) \cong y_\chi^{eq}(x)$
- For $x \gg x_F \Rightarrow y_\chi(x) \simeq y_\chi^{eq}(x_F) \leftarrow$ CONSTANT $\qquad \frac{x}{y_\chi^{eq}}\frac{dy_\chi}{dx} \simeq 0$




← In equilibrium

# Hot Relics




If $x \ll 1$

$$n_\chi^{eq} = \frac{\zeta(3)}{\pi^2} T^3 \times \left\{ \begin{matrix} \frac{3}{4} g & F \\ g & B \end{matrix} \right\} \tilde{g}$$

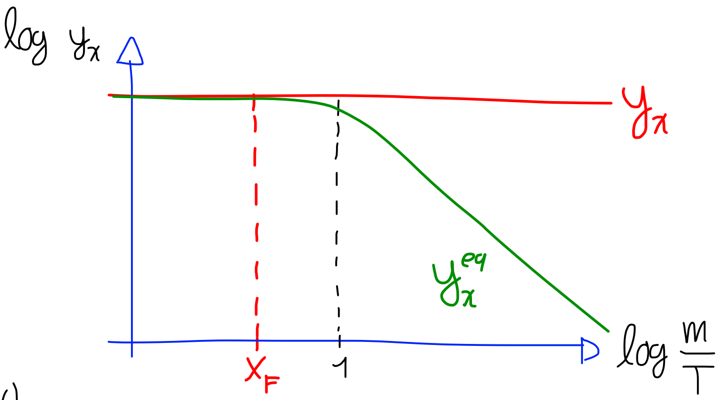
$$s = \frac{2\pi}{45} g_*^s(T)\, T^3$$

$\Rightarrow$ $Y_{eq} = \frac{n_{eq}}{s}$ depends weakly on $T$

$$Y_{eq} \simeq 0{,}28 \frac{\tilde{g}}{g_*^s(T)}$$

After freeze-out

$$Y(x > x_F) = Y(x_F) \simeq 0{,}28 \frac{\tilde{g}}{g_*^s(T_F)}$$




Relic abundance of neutrinos

$T_F \simeq 1 \text{ MeV} \rightarrow x_{F,\nu} \ll 1$

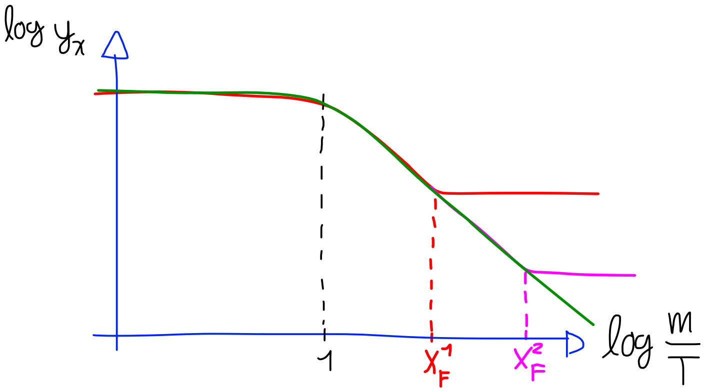
Today $\quad T_0^\nu \sim 10^{-3}\text{ eV} \ll m_\nu \sim O(\text{eV})$

$$\Rightarrow \rho_\nu^0 = m_\nu n_\nu^0 = m_\nu Y_\nu(x_{now})\, s^0 = m_\nu Y_\nu(x_F^\nu)\, s^0 \qquad s^0 \cong 3000 \text{ cm}^{-3}$$

$$\Omega_\nu h^2 = \frac{\rho_\nu^0 h^2}{\rho_{crit}^0} \simeq \frac{0{,}28\ \frac{3}{4} 2\ 3000\ \text{cm}^{-3} h^2\, m_\nu}{\underbrace{10{,}75}_{g_*^s(1\text{MeV})}\ (1{,}05\ 10^4 \text{ eV cm}^{-3} h^2)} \simeq \frac{m_\nu}{91 \text{ eV}}$$

Current bond $\quad m_\nu \lesssim 0{,}3$ eV

# Cold Relics




In this case

$$Y_{eq}(x) = \frac{1}{s}\, g \left(\frac{m_\chi T}{2\pi}\right)^{3/2} \exp\left(-\frac{m_\chi}{T}\right) \simeq 0{,}14\, \frac{g}{g_*^s}\, x^{3/2} e^{-x}$$

It has a x-dependence to compute $x_F$ one should solve the Boltzmann eq numerically

## Simplified method

$$\text{now} \rightarrow Y_\chi(T_0) \simeq Y_\chi(T_F) = \frac{n(x_F)}{s(x_F)} \simeq \frac{1}{s(x_F)} \frac{H(x_F)}{\langle\sigma v\rangle_{T_F}}$$

$$H(T_F) \propto T_F^2, \quad s(T_F) \propto T_F^3$$

$$\Omega_\chi h^2 = \frac{1}{\rho_c^0 h^{-2}} Y_\chi(T_0)\, m_\chi\, s^0 = \left(\frac{s^0}{\rho_c^0 h^{-2}}\right) \frac{H(T_F)\, m_\chi}{\langle\sigma v\rangle_{T_F}\, s(x_F)} \sim \frac{m_\chi}{T_F} \frac{g_*(T_F)^{1/2}}{g_*^s(T_F)} \frac{10^{-27}\ cm^3 s^{-1}}{\langle\sigma v\rangle_{T_F}}$$

To know $T_F$ we estimate

$$n_{eq}(x_F)\langle\sigma v\rangle_{T_F} = g\left(\frac{m_\chi T_F}{2\pi}\right)^{3/2} \exp\left(-\frac{m_\chi}{T_F}\right)\langle\sigma v\rangle_{T_F} \sim 1{,}66\, g_*^{1/2} \frac{T_F^2}{M_{Pl}}$$

$$\Longrightarrow \left(\frac{m_\chi}{2\pi T_F}\right)^{-1/2} e^{m_\chi/T_F} \simeq g\langle\sigma v\rangle_{T_F} \frac{M_{Pl}\, m_\chi}{g_*^{1/2}}\, 0{,}038 \simeq K \longleftarrow$$

we assume that $\langle\sigma v\rangle_{T_F}$ does not depend on $T_F$

$$\Longrightarrow \frac{m_\chi}{T_F} \simeq \log K + \frac{1}{2}\log\frac{m_\chi}{T_F} \simeq \log K + \frac{1}{2}\log(\log K) + \dots \quad \text{etc}$$

For example let's take $g \simeq 2$, $g_* \simeq 60$

$$\frac{m_\chi}{T_F} \sim 22 - \log\frac{m_\chi}{100\ GeV} \Longrightarrow \boxed{\Omega_\chi h^2 \sim \frac{3\ 10^{-27}\ cm^3 s^{-1}}{\langle\sigma v\rangle_{T_F}}}$$

- If $\langle\sigma v\rangle$ increases, $\Omega_\chi$ decreases and viceversa
- For $\Omega_\chi h^2 \sim 0{,}1$ (DM abundance) $\Longrightarrow \langle\sigma v\rangle \sim 3\ 10^{-26}\ cm^3 s^{-1} \sim 1$ pb  ($1\ s^{-1} = \frac{1}{3\ 10^{10}}\ cm^{-1}$)

**WIMP MIRACLE** It is an EW type of cross section for $m_\chi \sim$ EW scale !!

$$\langle\sigma v\rangle \sim \frac{c}{8\pi}\frac{\alpha^2}{m_\chi^2}, \quad c \sim O(1) \quad \Longrightarrow \langle\sigma v\rangle \simeq \left(\frac{100\ GeV}{m_\chi}\right)^2 23\ 10^{-27}\ cm^3 s^{-1}$$

For example, the WW production cross section at the LHC is
$\sigma_{WW} \sim 30$ pb

1 barn = $10^{-24}$ cm$^2$
1 pb = $10^{-36}$ cm$^2$

# BARYOGENESYS

There is an evident asymmetry between matter and antimatter
In cosmic rays $\phi_{\bar{p}}/\phi_p \sim 10^{-4}$ and all $\bar{p}$ are accounted for by $p + H \to 3p + \bar{p}$

$\to$ No primary antimatter in the galaxies

No spacial segregation between matter and $\overline{\text{matter}}$: since we don't see any $\gamma$-rays from such annihilation, the domain should be bigger than the observed Universe

If there was a symmetry $n_B = n_{\bar{B}}$ and their evolution was thermal, what would be their abundance?
They decouple at $\Gamma \sim \langle \sigma v \rangle n_B \sim \frac{n_B}{m_\pi^2} \simeq H = 1{,}66\, g_*^{1/2} \frac{T^2}{M_{Pl}}$ $\Rightarrow$ $\frac{m_p}{T_F} \simeq 42$ $\Rightarrow$ $T_f \simeq 22$ MeV

$$Y_B^0 \sim 10^{-20} \quad \Rightarrow \quad \text{Instead now} \quad \boxed{Y_B^0 = \frac{n_B^0}{s^0} \simeq \frac{\eta_B}{7} \sim 10^{-10}} \qquad \eta_B^0 = \frac{n_B^0}{n_\gamma^0}$$

$\Rightarrow$ The asymmetry in $B, \bar{B}$ MUST have happened BEFORE this epoch

At early times, the amount of asymmetry was very small
For $T \gg 1$ GeV

$$\eta_B = \frac{n_q^{eq} - n_{\bar{q}}^{eq}}{n_\gamma^{eq}} \simeq 6 \;\; 10^{-10}$$

To create such a tiny baryonic asymmetry, we need either asymmetric initial conditions or

the asymmetry has to be generated BARYOGENESIS $\exists$ NECESSARY ingredients
SAKHAROV CONDITIONS

**Baryon Number** the SM is invariant under the following transformation of the quarks

$$q_i \to e^{i\alpha} q_i \,, \qquad \bar{q}_i \to e^{-i\alpha} \bar{q}_i$$

where $\alpha$ is constant and $q_i$ are all the quarks
The conserved charge associated to this symmetry is the baryon number

$$B = \frac{1}{3} N_q - \frac{1}{3} N_{\bar{q}}$$

**Lepton Number** As before but for leptons $(e, \mu, \tau)$. It can be defined for each generation separately $L_e = N_e + N_{\nu_e} - N_{\bar{e}} - N_{\bar{\nu}_e}$, $L_\mu$, $L_\tau$
The single lepton numbers $L_i$ are violated by neutrino mixing

$$\nu_e \longleftrightarrow \nu_\mu \quad \text{etc}$$

We can define the total lepton number $L = L_e + L_\mu + L_\tau$
This is violated if neutrinos are Majorana particles
$\Rightarrow$ experimental search for neutrinoless double beta decay

$$n + n \to p + p + e^- + e^- \quad \Leftarrow \text{violates } L$$

# SAKHAROV CONDITIONS

- ## BARYON NUMBER VIOLATION $\not{B}$

Obviously, if we start from a $B, \bar{B}$ symmetric initial condition, we need $B$ and $\bar{B}$ violating processes to create a baryon asymmetry

Example $\quad \Gamma(X \to Y + B) \neq 0 \qquad (B(X) = B(Y) = 0)$

- ## C and CP (=T) VIOLATION $\not{C}$ & $\not{CP}$

$$\left.\left.\begin{array}{lll} C: & \text{charge conjugation:} & \text{exchange of particles and antiparticles} \\ P & \text{parity} & \vec{x} \to -\vec{x},\ t \to t \Rightarrow \begin{array}{l}\vec{v} \to -\vec{v}, \\ \vec{p} \to -\vec{p}\end{array},\ \vec{L} \to \vec{L} \end{array}\right\} CP \quad \begin{array}{lll} \\ \\ T: & \text{time inversion} & t \to -t. \end{array}\right\} CPT$$

Obtaining a $B, \bar{B}$ asymmetric final state from a symmetric one is possible only if $C$ and $CP$ are both violated, since such a state is not invariant under either $C$ or $CP$

Any Lorentz invariant QFT is invariant under the joint CPT transformation

This is not a problem since $T$ is violated by the expansion of the Universe

Example: $\quad \Gamma(X \to Y + B) \neq \Gamma(\bar{X} \to \bar{Y} + \bar{B}) \qquad \Leftarrow \quad C$ violation

$$\Gamma(X \to Y + B_L) \neq \Gamma(\bar{X} \to \bar{Y} + \bar{B}_R) \qquad \Leftarrow \quad CP \text{ violation}$$

- ## Departure from thermal equilibrium

In thermal eq $\mu_B = -\mu_{\bar{B}}$ (by $B + \bar{B} \to \gamma\gamma$ and $\mu_\gamma = 0$) and $\mu_B = 0$ because $B$ is not conserved

Moreover $m_B = m_{\bar{B}}$ by CPT invariance $\Rightarrow f_B = f_{\bar{B}} \Rightarrow n_B = n_{\bar{B}}$

The Standard Model alone satisfies all 3 conditions

- $\cancel{B}$ . 1) The modern view of the SM is to consider it as an Effective Field Theory valid only for energies smaller than some high scale $\Lambda$.
  Effects of the "new physics" at that scale are described by "higher dimensional operators" terms in $\mathcal{L}$ with Energy dimension $> 4$ (remember that $[\mathcal{L}]=4$)

$$\mathcal{L}^{EFT} = \mathcal{L}_{SM}^{d\leq 4} + \sum_i \frac{1}{\Lambda^n} \mathcal{O}^{d=4+n}$$

For example the Fermi interaction is such an operator with $n=2$ and $\Lambda \simeq m_W$ in the context of nuclear physics
For the SM, for $n=1$ there is only one operator

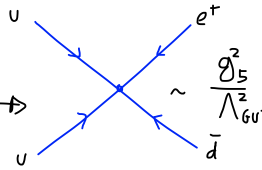
See Saw mechanism for neutrino masses

Violates $L$ $\longrightarrow$ $\frac{1}{\Lambda}\mathcal{O}^5 = \frac{1}{\Lambda}\left(\bar{\ell}_L^c H^+\right)\left(H \ell_L\right) \Rightarrow \frac{v^2}{\Lambda}\bar{\nu}_L^c \nu_L$ $\quad m_\nu \simeq \frac{v^2}{\Lambda}$

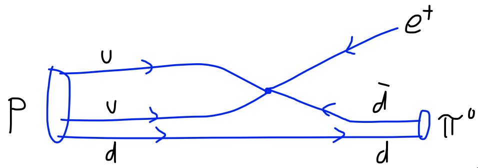
L and B are accidental symmetries of the SM

At $d=6$ there are many operators which violate $B$.
The experimental search for proton decay put a bound on $\Lambda \gtrsim 10^{15}$ GeV

like Fermi interaction $\longrightarrow$




$\sim \frac{g_5^2}{\Lambda_{GUT}^2}$




$p \to e^+ \pi^0 \Leftarrow$ Proton decay

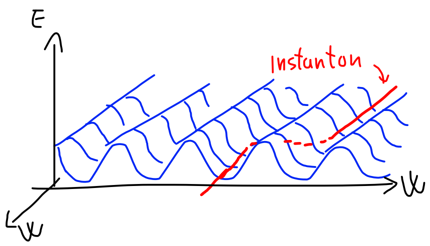
't Hooft $\rightarrow$ 2) Moreover, non perturbative interactions of the $SU(2)_L$ gauge symmetry (instantons) violate $B$ and $L$ (but conserve $B-L$). This may be important during the Electroweak Phase Transition (the epoch when $T \sim 10^2$ GeV) and the Universe passed from a $SU(2)_L \times U(1)_Y$ symmetric phase to the spontaneously broken phase

- $\cancel{C}$ Weak interactions violate $C$ maximally because distinguish the $L$ and $R$ chiralities $\left[\psi_L \xrightarrow{C} \bar{\psi}_R,\ \psi_R \xrightarrow{C} \bar{\psi}_L\right]$
- $\cancel{CP}$ CP is violated by a complex phase in some parameters of the theory (Cabibbo, Kobayashi, Maskawa mixing matrix) This shows up in some rare processes

- Departures from thermal equilibrium, as we saw, are easy to obtain in the history of the Universe. For example, the Electroweak Phase transition (EWPhT) could be such a departure if it was a first order one

# Electroweak Baryogenesys

The electroweak instantons (which violate B and L) are tunnelling processes through a potential barrier between gauge configurations with different topology. Since it's a tunnelling, its amplitude is exponentially suppressed. Also, B-L is conserved

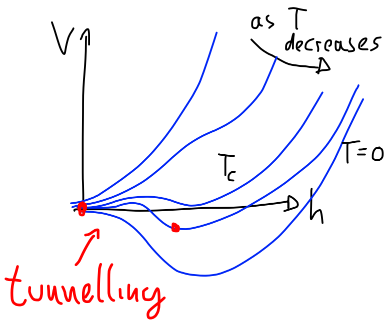



$$\Gamma \sim e^{-\frac{8\pi^2}{g^2}} \sim 10^{-162}$$

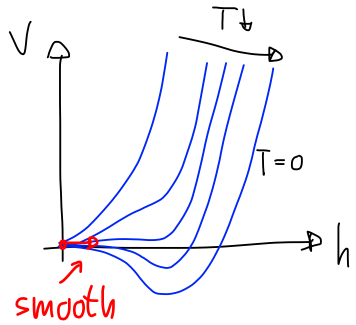
However, if $T$ is high enough ($T \gtrsim 300$ GeV) there is enough energy to pass OVER the barrier
sphalerons

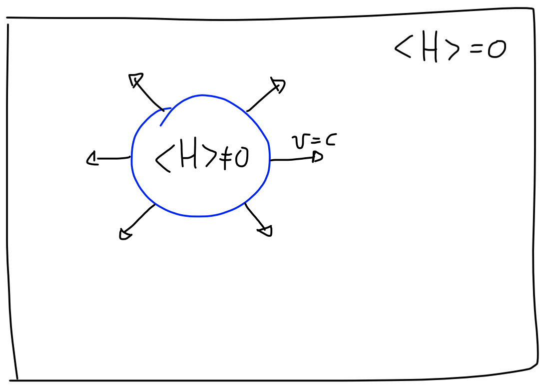
To have departure from thermal eq we want the Electroweak phase transition to be a first order one

1st order




2nd order




⟵ On the walls of the bubble we don't have thermal equilibrium
To have sufficient baryogenesis the walls need to be thick enough

However there are two problems

- In the SM the phase transition is not strong enough (one needs $m_H \lesssim 60$ GeV)
- The measured CP violation is too small to produce the observed asymmetry

In extensions of the SM (eg SUSY) this is still an open possibility

# High scale baryogenesis

Grand Unification Theories (GUT) are extensions of the SM in which the three gauge groups unify in a single simple group, for example $G_{GUT} = SU(5)$ or $SO(10)$ at a scale $\Lambda_{GUT} \sim 10^{16}$ GeV

$$\text{Eg} \qquad SU(5) \longrightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

These theories describe gauge couplings unification at $\Lambda_{GUT}$, quantization of quarks and lepton charges and their SM gauge representations, neutrino masses (see-saw), $\cancel{B}$ and $\cancel{L}$ $\Rightarrow$ p decay and relations among quark and lepton masses Moreover, they can offer a mechanism for baryogenesis

## Decay of heavy particles

Consider a heavy ($m_X \sim \Lambda_{GUT} \sim 10^{16}$ GeV) particle $X$ which can decay in two channels with different values of $B$, thereby violating $B$ conservation

$$① \quad \begin{matrix} X \to qq \\ \bar{X} \to \bar{q}\bar{q} \end{matrix} \quad \left(B = \tfrac{2}{3}\right) \qquad \text{and} \qquad ② \quad \begin{matrix} X \to \bar{q}\,\bar{\ell} \\ \bar{X} \to q\,\ell \end{matrix} \quad \left(B = -\tfrac{1}{3}\right)$$

By CPT invariance $m_X = m_{\bar{X}}$ and also the decay rates $\Gamma_X^{tot} = \Gamma_{\bar{X}}^{tot}$

Note Branching ratios are defined as $BR_i = \dfrac{\Gamma_X^i}{\Gamma_X^{tot}}$, where $i$ is a specific decay channel

$$\text{Assume} \quad \begin{matrix} BR_1^X = r & \Rightarrow & BR_2^X = 1 - r \\ BR_1^{\bar{X}} = \bar{r} & \Rightarrow & BR_2^{\bar{X}} = 1 - \bar{r} \end{matrix} \qquad \Leftarrow \text{Because } \sum_i BR_i = 1$$

If C and CP are violated then $r \neq \bar{r}$

In avarage, the mean net baryon number produced by the decay of $X, \bar{X}$ is

$$\Delta B_X = r\left(\tfrac{2}{3}\right) + (1-r)\left(-\tfrac{1}{3}\right) \qquad \Delta B_{\bar{X}} = \bar{r}\left(-\tfrac{2}{3}\right) + (1-\bar{r})\left(\tfrac{1}{3}\right)$$

If the initial conditions are symmetric in $X, \bar{X}$, the net $\Delta B$ that we produce by the decay of a $X, \bar{X}$ pair is

$$\varepsilon = \Delta B_X + \Delta B_{\bar{X}} = r - \bar{r}$$

Finally, we need departure from thermal equilibrium. In fact, in thermal equilibrium, the inverse processes would re-create as many $X, \bar{X}$ particles as had decayed, reabsorbing the net baryon asymmetry.

We want $\quad \tau_X > H^{-1} \Rightarrow$ otherwise $X$ would be cosmologically unstable

$\qquad\qquad\quad T \ll m_X \Rightarrow$ In this way there is not enough energy to produce again $X, \bar{X}$

The initial thermal abundance of $X, \bar{X}$ in equilibrium is the same

$$n_X = n_{\bar{X}} \simeq n_\gamma \quad \Leftarrow \text{ at } T \gg m_X$$

For $T < m_X$ $\quad n_X = n_{\bar{X}} \propto (m_X T)^{3/2} \exp\left(-\frac{m_X}{T}\right) \ll n_\gamma \quad \Leftarrow$ annihilation is inefficient because $n_X \ll n_\gamma$

$$\Gamma_{Decay} \simeq \Gamma_X \sim \alpha\, m_X \qquad\qquad H \sim g_*^{1/2} \frac{T^2}{M_{PL}}$$

Inverse decay is small because there is no energy

- If $\Gamma_D \ll H$ $(\Leftrightarrow \tau_X \gg t)$ then for $T \sim m_X$ $\;X, \bar{X}$ are $\sim$ stable and do not decrease in number $\Rightarrow$ equilibrium is not mantained and $X, \bar{X}$ become overabundant. This is the departure from thermal equilibrium

When $X, \bar{X}$ decay $(\text{at } \tau_X \sim t \Leftrightarrow \Gamma_X \sim H)$ they are much overabundant

$$n_X = n_{\bar{X}} \sim n_\gamma$$

- To have $m_X > T_X$ at $\Gamma_X \simeq H$ we need

$$\alpha\, m_X \sim g_*^{1/2} \frac{T_X^2}{M_{PL}} \Rightarrow T_X \sim \left(\frac{\alpha}{g_*^{1/2}} m_X M_{PL}\right)^{1/2}$$

$$\Rightarrow \boxed{m_X \gtrsim g_*^{-1/2}\, \alpha\, M_{PL} \sim \left(\frac{\alpha}{10^{-2}}\right) 10^{16}\ \text{GeV}} \sim \text{GUT}$$

- Each decay produces an asymmetry $\varepsilon$. So, the net baryonic density after all $X, \bar{X}$ decayed will be

$$n_B \simeq \varepsilon\, n_X \sim \varepsilon\, n_\gamma \sim \varepsilon\, g_*^{-1} s$$

- If there will be no more $\not{B}$ interactions, this relation will remain valid always. In particular, now

$$\eta = \frac{n_B^0}{n_\gamma^0} \sim \frac{n_B^0}{s^0} \sim \frac{\varepsilon}{g_*} = (r - \bar{r}) \frac{1}{g_*} \simeq 6\ 10^{-10}$$

this is a quite natural value for $\varepsilon$ because CP is weakly violated and because this can only appear in second order in perturbation theory